\[
\theta_p = \beta_1 = \tan(\frac{\sigma_1 - \sigma_x}{\tau_{xy}}) = 67.5^\circ
\]

Since \(\sigma_1 - \sigma_x > 0\) and \(\tau_{xy} > 0\), the angle \(\beta_1\) is on the first quadrant and therefore “atan” and “atan2” yield the same result.

\[
\theta_p = \beta_1 = \arctan2(\sigma_1 - \sigma_x, \tau_{xy}) = 67.5^\circ
\]

Since \(\sigma_1 - \sigma_x > 0\) and \(\tau_{xy} < 0\), the angle \(\beta_1\) is on the second quadrant.

“atan” gives an angle in the range \([-\frac{\pi}{2}, \frac{\pi}{2}]\):

\[
\theta_p = \beta_2 = \arctan(\frac{\sigma_1 - \sigma_x}{\tau_{xy}}) = -67.5^\circ
\]

“atan2” gives the angle in the correct quadrant (here the second quadrant):

\[
\theta_p = \beta_1 = \arctan2(\sigma_1 - \sigma_x, \tau_{xy}) = 112.5^\circ
\]

Both “atan” and “atan2” yield the same result.

Note that when using the expression,

\[
\tan(\theta) = \frac{\sigma_1 - \sigma_x}{\tau_{xy}}
\]

Find principal angle using \(
\tan(\theta) = \left(\frac{\sigma_1 - \sigma_x}{\tau_{xy}}\right)
\)
Find principal angle using \( \tan(2\theta) = \left( \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \right) \)

Since \( \frac{\sigma_x - \sigma_y}{2} < 0 \) and \( \tau_{xy} > 0 \), the angle \( \beta_1 \) is on the second quadrant.

"atan" gives an angle in the range \( [-\frac{\pi}{2}, \frac{\pi}{2}] \):

\[
\theta_{p2} = \frac{\beta_2}{2} = 0.5 \tan\left( \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \right) = -22.5^\circ
\]

"atan2" gives the angle in the correct quadrant (here the second quadrant):

\[
\theta_{p1} = \frac{\beta_1}{2} = 0.5 \tan2(2\tau_{xy}, \sigma_x - \sigma_y) = 67.5^\circ
\]

Since \( \frac{\sigma_x - \sigma_y}{2} < 0 \) and \( \tau_{xy} < 0 \), the angle \( \beta_1 \) is on the third quadrant.

"atan" gives an angle in the range \( [-\frac{\pi}{2}, \frac{\pi}{2}] \):

\[
\theta_{p2} = \frac{\beta_2}{2} = 0.5 \tan\left( \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \right) = 22.5^\circ
\]

"atan2" gives the angle in the correct quadrant (here the third quadrant):

\[
\theta_{p1} = \frac{\beta_1}{2} = 0.5 \tan2(2\tau_{xy}, \sigma_x - \sigma_y) = -67.5^\circ
\]

Note that when using the expression,

\[
\tan(2\theta) = \left( \frac{2 \tau_{xy}}{\sigma_x - \sigma_y} \right)
\]

Only "atan2" gives the result in the correct quadrant.