1. What are the positions of Urbana and Delhi in spherical coordinates \( r, \theta, \phi \) (use \( R_E = 6371 \) km)?

\[
\begin{align*}
\text{Urbana} & : \quad r = 6371 \text{ km} \quad \theta = -1.539 \text{ rad} \quad \phi = 0.871 \text{ rad} \\
\text{Delhi} & : \quad r = 6371 \text{ km} \quad \theta = 1.348 \text{ rad} \quad \phi = 1.071 \text{ rad}
\end{align*}
\]

2. What are the positions of Urbana and Delhi in \( x, y, z \) coordinates?

Convert spherical to rectangular coordinates:

\[
\begin{align*}
\text{Urbana} & : \quad x = 153 \text{ km} \quad y = -4870 \text{ km} \quad z = 4104 \text{ km} \\
\text{Delhi} & : \quad x = 1236 \text{ km} \quad y = 5454 \text{ km} \quad z = 3051 \text{ km}
\end{align*}
\]

3. How far is it from Urbana to Delhi in a straight line through the Earth?

\[ \| \vec{r}_U - \vec{r}_D \| = 10435 \text{ km.} \]

4. How far is from Urbana to Delhi along a great circle route?

\emph{Angle between} \( \vec{r}_U \) \emph{and} \( \vec{r}_D \) \emph{gives} 12227 km.

5. What is the maximum latitude reached as we travel along the great circle between Urbana and Delhi?

\emph{Take normal vector} \( \vec{n} = \vec{r}_U \times \vec{r}_D \) \emph{and find angle from} \( \hat{k} \), \emph{giving maximum latitude} 79.7\(^{\circ}\).
6. What is an initial tangent vector to the great circle route from Urbana to Delhi in the \( \hat{i}, \hat{j}, \hat{k} \) basis?

Take the cross product \( \vec{n} \times \vec{r}_U \) and normalize to obtain \( 0.2152 \hat{i} + 0.6333 \hat{j} + 0.7434 \hat{k} \).

7. What are the spherical basis vectors \( \hat{e}_r, \hat{e}_\theta, \hat{e}_\phi \) at Urbana, written in the \( \hat{i}, \hat{j}, \hat{k} \) basis?

Evaluate the explicit formulas:

\[
\begin{align*}
\hat{e}_r &= 0.0240 \hat{i} - 0.7644 \hat{j} + 0.6443 \hat{k} \\
\hat{e}_\theta &= 0.9995 \hat{i} + 0.0313 \hat{j} \\
\hat{e}_\phi &= 0.0202 \hat{i} - 0.6439 \hat{j} - 0.7648 \hat{k}
\end{align*}
\]

8. When we take off from Urbana to fly to Delhi, which compass direction should we initially head in? Is this compass direction maintained as we fly along the great circle?

The tangent vector is in the \( \hat{e}_\theta, \hat{e}_\phi \) plane. The basis vector \( \hat{e}_\phi \) points North, so find the angle between the tangent vector and \( \hat{e}_\phi \). This gives a compass bearing of 13.59° (clockwise from North). This is not maintained as we fly along a great circle.