TAM 212 – Dynamics

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Recap
• Rigid Bodies in Contact, Gears

Today
• Rigid Body Acceleration, Gears

Assume
\[ \mathbf{\dot{v}}_P = \mathbf{v}_C + \omega \times \mathbf{r}_{CP} \]
\[ \mathbf{\ddot{v}}_P = \mathbf{\ddot{v}}_C + \omega \times \mathbf{\dot{r}}_{CP} + \omega \times (\omega \times \mathbf{r}_{CP}) \]

For 2D x-y plane:
\[ \mathbf{\dot{r}}_P = \mathbf{r}_C + \mathbf{r}_{CP} \]
\[ \mathbf{\dot{v}}_P = \mathbf{v}_C + \omega z \mathbf{r}_{CP} \]
\[ \mathbf{\ddot{a}}_P = \mathbf{\ddot{a}}_C + \alpha_z \mathbf{r}_{CP} - \omega_z^2 \mathbf{r}_{CP} \]

stick = no relative velocity at contact
\[ \mathbf{\dot{v}}_A = \mathbf{\dot{v}}_B \quad v_{An} = v_{Bn} \]

slip = relative velocity at contact
\[ \mathbf{\dot{v}}_A \neq \mathbf{\dot{v}}_B \quad v_{An} = v_{Bn} \]

Gears:
\[ \frac{\omega_{1z}}{\omega_{2z}} = -\frac{r_2}{r_1} \]
Standard Sign Conventions

\[
\vec{v}_{P_1} = \vec{v}_{C_1} + \vec{\omega}_1 \times \vec{r}_{C_1P_1} \\
= 0 + (0, 0, \omega_{1z}) \times (r_1, 0, 0) \\
= (0, \omega_{1z} r_1, 0)
\]

\[
\vec{v}_{P_2} = \vec{v}_{C_2} + \vec{\omega}_2 \times \vec{r}_{C_2P_2} \\
= 0 + (0, 0, \omega_{2z}) \times (-r_2, 0, 0) \\
= (0, -\omega_{2z} r_2, 0)
\]

\[
\vec{v}_{P_1} = \vec{v}_{P_2}
\]

\[
\omega_{1z} r_1 = -\omega_{2z} r_2
\]

\[
\frac{\omega_{1z}}{\omega_{2z}} = -\frac{r_2}{r_1}
\]

Standard sign convention uses CCW as positive direction for \(\vec{\omega}\) and let the resulting sign of \(\vec{\omega}\) to determine the actual direction.

Alternative sign convention
Example

\[ \frac{\omega_{3Z}}{\omega_{1Z}} = \begin{cases} A. \frac{r_1}{r_3} \\ B. \frac{r_3}{r_1} \\ C. \frac{r_1 r_3}{r_2} \\ D. \frac{r_2}{r_1 r_3} \end{cases} \]

A. Positive sign
B. Negative sign
Example

\[
\begin{vmatrix}
\omega_{4Z} \\
\omega_{1Z}
\end{vmatrix} = \begin{array}{l}
A. \frac{r_1 r_2}{r_3 r_4} \\
B. \frac{r_1 r_4}{r_2 r_3} \\
C. \frac{r_2 r_3}{r_1 r_4} \\
D. \frac{r_1 r_3}{r_2 r_4} \\
E. \frac{r_3 r_4}{r_1 r_2}
\end{array}
\]

\[
\frac{\omega_{4Z}}{\omega_{1Z}} = \boxed{	ext{A. Positive sign}}
\]

B. Negative sign
Acceleration in contact (“no slip”)

1. Differentiate velocity expressions

\[ \frac{\omega_{2z}}{\omega_{1z}} = -\frac{r_1}{r_2} \]

\[ \omega_{1z} r_1 = -\omega_{2z} r_2 \]

2. Contacting points have equal tangential acceleration
**Gear Drives**

### Gears

- **Equation:** \( \omega_2 \frac{z_2}{z_1} = -\frac{r_1}{r_2} \)
- **Explanation:** Larger gear spins slower in opposite directions.

![Gears Diagram](image)

### Chains

- **Equation:** \( v_{P_1} = v_{P_2} = v_{P_3} \) - same speeds
- **Explanation:** Larger gear spins slower in the same directions.

![Chains Diagram](image)
Example

\[ \vec{a}_P = (a_{px}, a_{py}) \]

\[ r_1 = 2 \text{ m} \]
\[ r_2 = 4 \text{ m} \]
\[ \vec{\omega}_1 = 2\hat{k} \text{ rad/s} \]
\[ \vec{\alpha}_1 = -4\hat{k} \text{ rad/s}^2 \]

**What is the direction of \( \vec{a}_P \)?**

\[ |a_{px}| = \text{ A. } 1 \quad \text{B. } 2 \quad \text{C. } 4 \quad \text{D. } 8 \quad \text{E. } 16 \text{ (m/s}^2) \]

\[ |a_{py}| = \text{ A. } 1 \quad \text{B. } 2 \quad \text{C. } 4 \quad \text{D. } 8 \quad \text{E. } 16 \text{ (m/s}^2) \]
Example

Planetary gears

ring fixed

\( \omega_{SZ} = 2 \text{ rad/s} \)

What is the direction of \( \vec{v}_p \)?
Speed and torque are transmitted at the **contact point**. The contact point is between the gear teeth along a line that passes through the line of centers of the two gears. This is a simplified view of the complex interaction between two gears. In this view, adopted in TAM 212, the contact point between two pitch circles is “**not moving**” and the transferring of torque is ideal (without losses).