Recap
• Rotating Rigid Body Motion

Today
• Rigid Body Motion, Constraints

points → position, velocity, acceleration
bodies → angular velocity & acceleration

\[ \vec{v}_{P/C} = \text{velocity of P relative to C} \]
\[ = \vec{v}_P - \vec{v}_C \]

\[ \vec{v}_{P/C} = \vec{\omega} \times \vec{r}_{CP} \]

\[ \vec{v}_P = \vec{v}_C + \vec{\omega} \times \vec{r}_{CP} \]

relative velocity equation

See “slide” animation in #rkg-fd

Works for any two points on RB:

\[ \vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ} \]
Example

\[ \vec{v}_D = \vec{v}_G + \vec{ω} \times \vec{r}_{GD} \]

\[ \vec{v}_D = \vec{v}_C + \vec{ω} \times \vec{r}_{CD} \]
Example

Which point is Q? Point A, B, C, or D
Which point is Q? Point A, B, C, or D
Example

Which point cannot be Q?
Point B, C, D, or E
(don’t choose A which is a trivial solution)
Constrained Motion

\[ \vec{v}_E = \hat{i} + 7\hat{j} \text{ m/s} \]

\[ \vec{r}_{AE} = 4\hat{i} - \hat{j} \text{ m} \]

Point A constrained to the 45° slot
Two methods for constrained velocities

1. Which way can it move?

\[ \Rightarrow \hat{u}_1 \Rightarrow \vec{v} = v_u \hat{u}_1 \]

2. Which way can’t it move?

\[ \Rightarrow \hat{u}_2 \Rightarrow \vec{v} \cdot \hat{u}_2 = 0 \]
1. Which way **can** it move?

\[ \vec{v}_E = \hat{i} + 7\hat{j} \text{ m/s} \]

\[ \vec{r}_{AE} = 4\hat{i} - \hat{j} \text{ m} \]

Point A constrained to the 45° slot
2. Which way can’t it move?

\[ \vec{v}_E = \hat{i} + 7\hat{j} \text{ m/s} \]

\[ \vec{r}_{AE} = 4\hat{i} - \hat{j} \text{ m} \]

Point A constrained to the 45° slot
Constrained Motion

\[ \vec{v}_E = 3\hat{i} + 12\hat{j} \text{ m/s} \]

\[ \vec{r}_{AE} = 4\hat{i} - \hat{j} \text{ m} \]

Point A constrained to the 30° slot

Which direction is \( \vec{\omega} \)?

A. CW
B. CCW
C. neither
D. can’t tell

Which direction is \( \vec{v}_A \)?

A. \( \rightarrow \)
B. \( \leftarrow \)
C. not moving
D. can’t tell
Example

\[ OP = 3\sqrt{2} \text{ m} \]
\[ PQ = 5 \text{ m} \]
\[ \vec{\omega}_1 = 4\hat{k} \text{ rad/s} \]

1. Predict
\[ \vec{\omega}_2 = \omega_{2z}\hat{k} \]
Slider-crank mechanism, arrangement of mechanical parts designed to convert straight-line motion to rotary motion, as in a reciprocating piston engine, or to convert rotary motion to straight-line motion, as in a reciprocating piston pump.
Example

$OP = 3\sqrt{2}$ m

$PQ = 5$ m

$\vec{\omega}_1 = 4\hat{k}$ rad/s

2. Calculate

3. Reflect  Does this make sense?
What is $\vec{v}_C$?

1. Predict

$\vec{r}_{AC} = (2, 1)$ m

$\vec{r}_{BC} = (1, 3)$ m

$\vec{v}_A = (0, 2)$ m/s

$\vec{v}_B = (1, 0)$ m/s