TAM 212 Worksheet 11: Centripetal Acceleration and Weight

At this moment you are at rest relative to the surface of the earth. The purpose of this worksheet is to investigate your motion \textit{in the reference frame of the earth}. In what follows, you will assume that the earth is perfectly spherical, and you will treat yourself as a particle. Figure 1 shows a schematic diagram of the system in question. Let the (non-rotating) \( xy \)-coordinate system have its origin \( O \) at the center of the earth, and take the positive \( y \)-axis to pass through the North Pole, as shown. Note that this reference frame is not truly an inertial frame, but to a very good approximation it can be treated like one.

Figure 1: Schematic diagram of the earth and you, with relevant geography labeled.
Vector Analysis

1. The way we have chosen the coordinate axes, the earth rotates about the $y$-axis with constant angular velocity $\vec{\omega} = \omega \hat{j}$, where $\omega > 0$. What are the numerical values of $\omega$ in rev/min and rad/s?

2. As the earth rotates about the $y$-axis, your trajectory (in the reference frame of the earth) is a circle of radius $r$. Label the center $C$ of this circle, as well as the distance $r$, in Figure 1.

3. Recall that the acceleration $\vec{a}$ of an object moving in a circle of radius $r$ with constant angular velocity $\omega$ points toward the center of the circle (i.e., it is centripetal). Draw your acceleration $\vec{a}$ in Figure 1 at the instant shown, placing the tail of the vector at your position. Below, express $\vec{a}$ in terms of $\omega$ and $r$ in the given Cartesian basis.

4. Label the radius of the earth $R_E$ and your latitude $\phi$ in Figure 1. Below, express $r$ in terms of $R_E$ and $\phi$. Use your answer to express $\vec{a}$ from Question 3 in terms of $\omega$, $R_E$, and $\phi$.

5. Draw the force of gravity $\vec{F}_G$ on you in Figure 1. Below, express $\vec{F}_G$ in the given Cartesian basis in terms of your mass $m$, the acceleration $g$ due to gravity on the surface of the earth, and $\phi$. Note that $g > 0$. 
6. Note that $\vec{a}$ and $\vec{F}^G$ do not point in the same direction. This means that there must be an additional force $\vec{F}$ acting on you. Find $\vec{F}$ in the given Cartesian basis in terms of $m$, $g$, $\omega$, $R_E$, and $\phi$.

7. Express the unit vectors $\hat{u}$ and $\hat{v}$ in the given Cartesian basis in terms of $\phi$.

8. Use your answers to Questions 6 and 7 to find the normal $F_u$ and tangential $F_v$ components of $\vec{F}$.

9. Draw $\vec{F}$ in Figure 1. Toward which geographical landmark does the tangential component point?
Measured Weight as a Function of Latitude

10. Plot your weight $W$ (as measured by a weighing scale) as a function of $\phi$ in Figure 2 for $\phi \in [-\pi/2, \pi/2]$. Label the maximum and minimum values of $W$ on the vertical axis in terms of $m$, $g$, $\omega$, and $R_E$.

[Hint: Weighing scales do not measure the gravitational force directly...what do they actually measure?]

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Figure 2: Plot of measured weight $W$ versus latitude $\phi$.
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11. Based on your plot from Question 10, at which geographical landmark(s) is your measured weight maximum? Minimum? What are these maximum and minimum values?

12. What is the fractional difference between your true weight and your weight as measured at a given latitude $\phi$? [Note: fractional difference = (true weight - measured weight)/(true weight)] Leave your answer in terms of $g$, $\omega$, and $R_E$. 


13. The latitude of Urbana, IL is $40.1097^\circ$. Label this point on your plot in Figure 2. What is the fractional difference between your true weight and your weight as measured in Urbana? Recall that the radius of the earth is $R_E = 6371$ km.

14. If a person’s mass is 165 lbm, what is that person’s weight, as measured in Urbana, in lbf? [Note: 1 slug = 32.2 lbm, 1 lbf = 1 slug-ft/s$^2$]

15. By what factor should you multiply your weight (as measured in Urbana) to find your true weight?
Challenge: Minimum Necessary Coefficient of Static Friction

You will now investigate whether dry friction is enough to provide $F_v$. In particular, you will find the minimum coefficient of static friction required to keep objects from sliding on the surface of the earth.

16. Using the relation $F_v < \mu_s F_u$, find $\mu_{\text{min}}(\phi)$ such that, if $\mu_s > \mu_{\text{min}}(\phi)$, an object will not slip on the surface of the earth. Express your answer in terms of $g$, $\omega$, and $R_E$.

17. It can be shown that $\mu_{\text{min}}(\phi)$ attains its maximum value at $\phi = \pi/4$. Find this maximum value in terms of $g$, $\omega$, and $R_E$.

18. What is the numerical value of your answer to Question 17?
19. Based on your answer to Question 18, is it plausible that dry friction is providing the force required to keep objects from sliding on the surface of the earth? [Hint: The coefficient of static friction between ice and ice is 0.02.]

20. The angle $\theta$ that $\vec{F}$ makes with the normal direction defined by $\hat{v}$ varies with latitude $\phi$. What is its maximum value? [Hint: Use your answer to Question 18.]