The aim of this worksheet is to understand how Guerlain Chicherit performed his backflip in a Mini Cooper Countryman in Tignes, France, in February, 2013. The press release from BMW (owners of the Mini brand) said:

The backflip had previously been attempted by other drivers. But Chicherit has now become not only the first automotively-propelled artist to execute a perfect landing, he also did it “unassisted” — in other words, without the aid of a special ramp with moving elements to boost the cars rotational movement. For his take-off, Chicherit used a static ramp that fits the same template as a quarterpipe on a freestyle course. With the ideal ramp breakover angle in place, Chicherit needed only two other things to record a successful attempt: an extremely light touch with the accelerator and a MINI with a suitably buoyant spring in its step.

The question is whether the curvature of the ramp is sufficient to produce the car rotation, or whether the car needs to have hydraulic actuators in the wheels or something similar to make it flip (a “suitably buoyant spring”?). We will proceed in several steps:

- Estimate the trajectory of the car, regarding it as a single point. This will determine the duration $t_J$ of the jump.
- Find the angular velocity $\omega_J$ required to rotate the correct angle during the jump.
- Find the angular velocity $\omega_R$ that would be produced solely by the ramp curvature.
- Compare the two angular velocities above.

![Diagram of the ramp and car](https://www.press.bmwgroup.com/global/presdetail.html?title=leap-land-lap-up-the-plaudits-mini-shows-snowboarders-how-it%E2%80%99s-done-with-the-perfect-backflip&outputChannelId=6&id=T0137069EN&left_menu_item=node__5128)
A photo of the car mid-jump is shown above. Various dimensions have been drawn and measured in Photoshop, using the length of the car as a reference and data from http://miniusa.com. Note that the length of \( \vec{v}_0 \) above is not meaningful, and only the direction of this vector is known.
Jump Analysis

In this page, we will consider the car as a point mass.

1. The car launched from the origin $O$ at time $t = 0$ with initial velocity $\vec{v}_0$. Draw a free body diagram and find the acceleration of the car while in the air ($g = 9.81 \text{ m/s}^2$). Write your answer explicitly in $\hat{i}$ and $\hat{j}$ components. Neglect the air drag resistance.

\[ \vec{a} = -g \hat{j} \text{ m/s}^2 \]
\[ \vec{v} = \vec{v}_0 - gt \hat{j} \text{ m/s} \]
\[ \vec{r} = \vec{r}_0 + \vec{v}_0 t - \frac{1}{2} gt^2 \hat{j} \]
\[ = v_0 t \cos \theta_0 \hat{i} + v_0 t \sin \theta_0 \hat{j} - \frac{1}{2} gt^2 \hat{j} \]
\[ = (v_0 t \cos \theta_0) \hat{i} + \left( v_0 t \sin \theta_0 - \frac{1}{2} gt^2 \right) \hat{j} \text{ m.} \]

2. From the acceleration of the car while in the air, integrate twice to obtain the position vector $\vec{r}(t)$ of the car in terms of the unknown initial speed $v_0$. Write your answer explicitly in $\hat{i}$ and $\hat{j}$ components.

3. Sketch the path of the car from $O$ to $Q$ on the diagram.

*This is a parabola starting at $O$, peaking somewhere near the car’s location in the image, and finishing at $Q$.\*
4. The car landed at $Q$, which means that $\vec{r}(t_J) = \vec{r}_Q$, where $t_J$ is the duration of the jump. Solve for $t_J$ and $v_0$.

\[ v_0 t_J \cos \theta_0 = L \quad \Rightarrow \quad v_0 t_J = \frac{L}{\cos \theta_0} \]

\[ v_0 t_J \sin \theta_0 - \frac{1}{2} g t_J^2 = H \]

\[ \frac{L \sin \theta_0}{\cos \theta_0} - \frac{1}{2} g t_J^2 = H \]

\[ t_J = \sqrt{\frac{2}{g} (L \tan \theta_0 - H)} \]

\[ t_J = 2.47 \text{ s}, \]

where we take the positive time solution. Then:

\[ v_0 = \frac{L}{t_J \cos \theta_0} \]

\[ = 15.3 \text{ m/s}. \]
From now, we start to consider the car as a rigid body.

5. The car landed in a horizontal configuration with wheels down. What is the angle $\theta_J$ that it rotated through during the jump?

*The car is rotating counterclockwise, so $\theta_J = 360^\circ - \theta_0 = 290^\circ$.*

6. Assuming a constant angular velocity $\omega_J$ during the jump, what is $\omega_J$?

$$\omega_J = \frac{\theta_J}{t_J} = 2.05 \text{ rad/s}$$

7. Sketch the tangential/normal basis for the car at the launch point $O$, just before leaving the ramp, on the figure.

*\hat{e}_t$ is in the direction $\vec{v}_0$, $\hat{e}_n$ is towards the center of the osculating circle.*

8. What is the curvature $\kappa$ of the car’s path up the ramp? Determine the angular velocity $\omega_R$ of the car due to the ramp curvature.

*Curvature is $\kappa = 1/R = 0.1 \text{ m}^{-1}$, and the angular velocity is $\omega_R = v_0\kappa = 1.67 \text{ rad/s}$.*

9. Is the ramp curvature sufficient to explain the jump rotation (within a reasonable margin of experimental error)? What are the possible sources of this discrepancy? What radius of curvature for the ramp would exactly match the jump angular velocity?

*It is plausible that the ramp is providing nearly all of the required angular velocity. Perhaps a small additional angular velocity may need to be provided by the car, or spinning the wheels while in the air may produce some small additional rotation. A ramp with radius of curvature of $v_0/\omega_J = 7.45 \text{ m}$ would be exactly correct, given the numbers used here.*
10. In main section, we assumed that $\omega_R$ (orbital angular velocity during the circular motion) is equal to $\omega_J$ (spinning angular velocity during the jump). Discuss the validity of this assumption.

In this case, “orbital angular velocity” and “spinning angular velocity” are coupled with the same angular velocity when the car is inside the ramp. We simply assumed this spinning angular velocity also remains the same during the jump as $\omega_J$, because of inertia.

In general, spinning and orbital angular velocity are different; such as planetary gears, rotation and revolution of the earth. Also, think about the difference between surveillance satellites and geosynchronous satellites.

11. What was the speed of the car when it hit the ground at $Q$?
Note: Use values from main section of worksheet.

$$
\vec{v}(t_J) = v_0 \cos \theta_0 \hat{i} + v_0 \sin \theta_0 \hat{j} - gt_J \hat{j}
$$

$$
= 5.23\hat{i} - 9.81\hat{j} \text{ m/s}
$$

$$
\vec{v}(t_J) = 11.1 \text{ m/s}
$$

$$
= 24.9 \text{ mph}
$$

12. The car landed rear-wheels first. What was the speed of the rear wheel impact site when it hit the ground? Assume that the rear wheel impact site is 2 m back and 0.5 m down from the center of rotation, and that the center of rotation has the velocity found in the previous question.
Note: Use values from main section of worksheet.

Taking $M$ to be the center of rotation and $W$ to be the point that impacts, $\vec{r}_{MW} = -2\hat{i} - 0.5\hat{j}$ and:

$$
\vec{v}_W = \vec{v}_M + \omega_J \hat{k} \times \vec{r}_{MW}
$$

$$
= 6.26\hat{i} - 13.9\hat{j} \text{ m/s}
$$

$$
\vec{v}_W = 15.3 \text{ m/s}
$$

$$
= 34.1 \text{ mph}
$$

13. Which way is the binormal basis vector $\hat{e}_b$ pointing, just before and after leaving the ramp? What is the torsion $\tau$ of the car’s path up the ramp?

Before leaving the lamp, $\hat{e}_b$ is out of the page. It changes the direction discontinuosly to the opposite direction after passing $O$. Torsion is $\tau = 0 \text{ m}^{-1}$ (2D path).