TAM 212 Worksheet 4

Solutions

The worksheet is concerned with the design of the loop-the-loop for a roller coaster system.

Old loop design: The first generation of loops was circular, as shown below.

![Old Loop Design](image1)

New loop design: The modern loop has evolved into the tear-drop track shape pictured below.

![New Loop Design](image2)
1. Briefly predict what issues might arise due to a circular loop, and how a tear-drop loop might resolve those problems.

*After a train enters this circular loop, the passengers who will feel this as a sudden jerk comes from the centripetal acceleration. Rarely, this jerk even causes a neck injury.*

**Circular Track**

The first generation of loops were circular, as illustrated on Page 1. Although not strictly accurate, we’ll assume for this section that the roller coaster train maintains a constant speed as it travels along the track.

2. For the circular loop, plot the curvature $\kappa$ of the track as a function of $s$, the total distance covered. Label the important points on the vertical axis in terms of the loop radius $R$. Note that $s_1$ and $s_2$ denote the point where the train enters and leaves the loop, respectively.

3. Plot the tangential $a_t$ and normal $a_n$ component of the train’s acceleration $\vec{a} = a_t \hat{e}_t + a_n \hat{e}_n$ as a function of $s$, the total distance covered. Label the important points on the vertical axis in terms of the train speed $v$ and the loop radius $R$. 

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![Curvature Plot](image1)

![Acceleration Components Plot](image2)
4. The circular loop design, popular in the earliest inversion roller coasters, was in fact responsible for many broken bones and neck injuries. Why do you think this may have occurred?

The normal component of the acceleration will suddenly jump from zero on the straight-line segments to \( v^2/R \) on the loop. This will not be safe for passengers, who will be jerked upwards as a result of the sudden change in acceleration.

5. Now plot the \( a_z \) (upwards, \( \hat{k} \)) component of the train’s acceleration as a function of \( s \), the total distance covered. Label all significant points on the vertical axis.

![Diagram of acceleration vs. distance](image)

**Tear-Drop Track**

The modern loop has evolved into the teardrop-like shape as exhibited by the roller coaster on Page 1. Also, we’ll assume for this section that the roller coaster train maintains a constant speed.

6. To reduce risk of injury, the teardrop shape for the loop shown below is now commonly employed. How do you think this shape helps to give passengers a smoother ride?

![Diagram of teardrop loop](image)

*The curvature of the loop appears to vary more continuously, starting from zero and increasing continuously as we approach the top, and then decreasing continuously back to zero as we exit the curve. The continuous variation in curvature means there will not be a jump in the acceleration.*
7. A curve for which the curvature varies linearly with the distance covered is known as a *clothoid* or *Euler spiral*. The teardrop shape above closely resembles two symmetric clothoids, in which the curvature increases linearly with $s$ after the train enters the curve, reaches a maximum value of $1/R$ at the top of the curve, and then linearly decreases back to zero when the train exits the loop. For such a loop, plot the curvature $\kappa$ as a function of $s$. Note that $s_A$ and $s_E$ denote the point where the train enters and leaves the loop (as illustrated on the next page).

8. For the teardrop loop, sketch the normal and tangential acceleration (labeling each) at the five points demarked below.

*Normal acceleration shown; there is no tangential acceleration since the train’s speed is assumed constant.*

9. For the teardrop loop, plot the tangential $a_t$ and normal $a_n$ component of the train’s acceleration $\vec{a} = a_t \hat{t} + a_n \hat{n}$ as a function of $s$, the total distance covered. Label the important points on the vertical axis in terms of the train speed $v$ and the radius or curvature at the top of the loop $R$. 

\[
\begin{align*}
    a_t &\quad (\text{left}) \\
    a_n &\quad (\text{right})
\end{align*}
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10. Now plot the $a_z$ (upwards, $\hat{k}$) component of the train’s acceleration on the teardrop loop as a function of $s$, the total distance covered. Where is the acceleration in the $\hat{k}$–direction felt by the passengers largest? How large is this?

![Diagram showing $a_z$ as a function of $s$]

The acceleration experienced by the passengers is largest at the top of the loop (magnitude $v^2/R$, directed downwards).
Challenge: Energy Analysis

11. As you have likely experienced on roller coasters, the speed does not stay constant as you traverse the loop. Instead, the speed decreases as you travel up the curve and increases as you move back down the track. Use conservation of energy to calculate the expected speed at the top of a 25 m tall loop when the initial speed is 10 m/s, where kinetic energy is $KE = \frac{1}{2}mv^2$ and potential energy is $PE = mgh$.

12. Based on your energy analysis, is constant speed a reasonable assumption? How would your analysis in the first two sections change if the velocity becomes dependent on track position?

13. What other physics would you include to make your analysis even more accurate?