1. Below is a diagram showing the Earth’s position over time as it revolves around the sun. On this diagram, sketch what you think the path of the moon looks like. The precise scaling is not important, but try and get the correct “shape” of the orbit. Don’t spend too long on this, but just give it your best shot.

See Question 7.

2. The earth revolves around the sun in the counterclockwise direction, completing one full revolution about every 365 days. In reality the orbit is elliptical (with an eccentricity of 0.0167), but we’ll pretend that the orbit is a perfect circle. How many revolutions per day does the earth make around the sun? What is the angular velocity \( \omega_{SE} \) (radians/day) of the earth around the sun?

\[
\frac{1}{365} \text{ revolutions per day} \quad \omega_{SE} = \frac{2\pi}{365} \text{ rad/day}
\]

3. With the sun at the origin, write the position vector for the earth as a function of time in the Cartesian and polar bases. You can leave your answer in terms of the earth’s angular position \( \theta_E \) and the distance between the sun and the earth \( R_{SE} \).

\[
\text{Cartesian: } \vec{r} = R_{SE} \cos \theta_E \hat{i} + R_{SE} \sin \theta_E \hat{j} \\
\text{polar: } \vec{r} = R_{SE} \hat{e}_r
\]

4. What is the expression for the velocity and acceleration of the earth in the Cartesian basis? In the polar basis? Leave your answer in terms of \( \theta_E \) and \( \dot{\theta}_E \), recalling that \( \dot{\theta}_E = \omega_{SE} \).

\[
\text{Cartesian: } \\
\vec{v} = -R_{SE} \dot{\theta}_E \sin \theta_E \hat{i} + R_{SE} \dot{\theta}_E \cos \theta_E \hat{j} \\
\vec{a} = -R_{SE} \ddot{\theta}_E \cos \theta_E \hat{i} - R_{SE} \dot{\theta}_E^2 \sin \theta_E \hat{j}
\]

\[
\text{polar: } \\
\vec{v} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta = R_{SE} \dot{\theta}_E \hat{e}_\theta \\
\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \hat{e}_\theta = -R_{SE} \ddot{\theta}_E^2 \hat{e}_r
\]

5. On the diagrams provided below, plot the radial and angular velocity and acceleration of the earth. 

See solution plot on the last page.

6. We all know that the moon revolves around the earth. However, when observed from the vantage point of a fixed sun, the path of the moon can be surprising. The moon orbits the earth in the counterclockwise direction, completing one full revolution every 28 (solar) days at a distance of \( R_{EM} = 0.0026 R_{SE} \). Write down in the Cartesian basis the position vector of the moon from the earth, \( \vec{r}_{EM} \), in terms of \( R_{EM} \) and the moon’s angular position \( \theta_M \). Then write down the position vector of the moon from the sun, \( \vec{r}_{SM} \), in the Cartesian basis.
\[ \vec{r}_{EM} = R_{EM} \cos \theta_M \hat{i} + R_{EM} \sin \theta_M \hat{j} \]
\[ \vec{r}_{SM} = (R_{SE} \cos \theta_E + R_{EM} \cos \theta_M) \hat{i} + (R_{SE} \sin \theta_E + R_{EM} \sin \theta_M) \hat{j} \]

7. Sketch the trajectory of the moon, as the earth travels around the sun, on the diagram provided below. The diagram shows the position of the earth at about 28-day intervals. What object does the moon appear to be orbiting? How does your diagram compare to your diagram that you drew in Question 1?

See last page for plots. The moon appears to orbit the sun.

8. What are the expressions for the velocity and the acceleration of the moon in the Cartesian basis? Leave your answer in terms of \( \theta_E, \dot{\theta}_E, \theta_M, \) and \( \dot{\theta}_M, \) where \( \dot{\theta}_M \) is the angular velocity of the moon around the earth.

\[ \vec{v}_M = -\left(\dot{\theta}_E R_{SE} \sin \theta_E + \dot{\theta}_M R_{EM} \sin \theta_M\right) \hat{i} + \left(\dot{\theta}_E R_{SE} \cos \theta_E + \dot{\theta}_M R_{EM} \cos \theta_M\right) \hat{j} \]
\[ \vec{a}_M = -\left(\ddot{\theta}_E^2 R_{SE} \cos \theta_E + \ddot{\theta}_M^2 R_{EM} \cos \theta_M\right) \hat{i} - \left(\ddot{\theta}_E^2 R_{SE} \sin \theta_E + \ddot{\theta}_M^2 R_{EM} \sin \theta_M\right) \hat{j} \]

9. We’ll now see how the radial and angular components of the velocity and acceleration are different for the moon’s trajectory in comparison to the earth’s trajectory. Making use of the expressions in the question above, fill out the chart below for the radial and angular components of the moon’s velocity and acceleration for the given orientations:

<table>
<thead>
<tr>
<th>( \theta_E )</th>
<th>( \theta_M )</th>
<th>( v_r )</th>
<th>( v_\theta )</th>
<th>( a_r )</th>
<th>( a_\theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>(-\theta_M R_{EM})</td>
<td>(-\theta_E R_{SE})</td>
<td>(-\theta_E^2 R_{SE})</td>
<td>0</td>
</tr>
<tr>
<td>( \pi/2 ) rad</td>
<td>( \pi ) rad</td>
<td>(-\theta_M R_{EM})</td>
<td>( \theta_E R_{SE} - \theta_M R_{EM})</td>
<td>(-\theta_E^2 R_{SE})</td>
<td>( \theta_M R_{EM})</td>
</tr>
<tr>
<td>( \pi ) rad</td>
<td>(-\pi/2 ) rad</td>
<td>0</td>
<td>( \theta_E R_{SE} - \theta_M R_{EM})</td>
<td>(-\theta_E^2 R_{SE})</td>
<td>( \theta_M R_{EM})</td>
</tr>
</tbody>
</table>

10. Using the table above as a guide, sketch the angular and radial components for the velocity and acceleration of the moon on the plots that you used for the earth’s velocity and acceleration in Question 5.

See solution plot on the last page.

11. (Bonus question) How would the trajectory of the moon be different if, everything else staying the same, the distance between the earth and the moon were suddenly increased by a factor of 100?

See solution plot on the last page.
\[ V_\| (10^{-3} \text{ AU/day}) \]
\[ \theta \| (10^{-2} \text{ AU/day}) \]
\[ a_\theta (10^{-4} \text{ AU/day}^2) \]
\[ V_\theta (10^2 \text{ AU/day}) \]
\[ \theta \| (10^{-4} \text{ AU/day}^2) \]

\[ \text{earth} \]
\[ \text{moon} \]

\[ \text{moon, } R_{\text{moon}} = R_{\text{earth}} \times 100 \]