

09-15-17

Lecture #8

i Clicker:

Cylindrical coords

$$\vec{r}_{op} = r \hat{e}_r + z \hat{k}$$

Q1: The unit vector \hat{e}_r is in the direction of the vector

$$\text{proj } (\vec{r}_{op}, \hat{i}) + \text{proj } (\vec{r}_{op}, \hat{j})$$

(a) true

(b) false

$$\vec{r}_{op} = \alpha \hat{i} + \beta \hat{j} + z \hat{k}$$

Q2: The above vector $\text{proj } + \text{proj}$ is always a unit vector?

(a) true

(b) false

Consider the two vectors



Q3: Which of the following statements is definitely true

(a) $\vec{x} \cdot \vec{y} < 0$

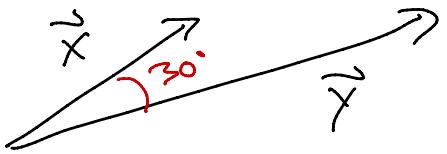
(b) $\vec{x} \cdot \vec{y} > 0$

(c) $\vec{x} \cdot \vec{x} = \vec{x} \cdot \vec{y}$

(d) $\vec{x} \cdot \vec{y} = 0$

$$\vec{x} \cdot \vec{y} = \|\vec{x}\| \cdot \|\vec{y}\| \cos \theta$$

Q4: "Same as Q3"

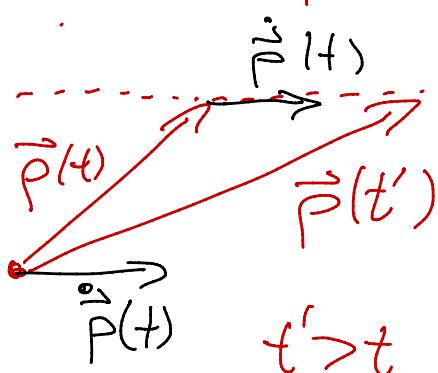


- (a) $|\vec{x} \cdot \vec{y}| = \|\vec{x}\| \cdot \|\vec{y}\|$
- (b) $|\vec{x} \cdot \vec{y}| \geq \|\vec{x}\| \cdot \|\vec{y}\|$
- (c) $|\vec{x} \cdot \vec{y}| < \|\vec{x}\| \cdot \|\vec{y}\| \quad \checkmark$
- (d) $\vec{x} \cdot \vec{y} < -\|\vec{x}\| \cdot \|\vec{y}\|$

Q5: "Which of the following...".

- (a) $\vec{x} \cdot \vec{y} > \|\vec{x} \times \vec{y}\| \quad \checkmark$
 - (b) $\vec{x} \cdot \vec{y} < \|\vec{x} \times \vec{y}\|$
 - (c) there is not enough information to answer (a) or (b)
- $\|\vec{x} \times \vec{y}\| = \|\vec{x}\| \cdot \|\vec{y}\| \sin \theta \quad \times$

Q6: Consider the path given by the position vector $\vec{p}(t)$.



Is the statement

$$\|\text{proj}(\vec{p}(t), \vec{p}(t'))\| \geq 0$$

(a) true

(b) false. ?

pathological case

$$\vec{p} = 0$$

Position, velocity, and acceleration

$$\vec{p}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$\vec{v}(t) = \dot{\vec{p}}(t) = \dot{x}(t)\hat{i} + \dot{y}(t)\hat{j} + \dot{z}(t)\hat{k}$$

$$\vec{a}(t) = \ddot{\vec{p}}(t) = \ddot{x}(t)\hat{i} + \ddot{y}(t)\hat{j} + \ddot{z}(t)\hat{k}$$

In polar coords and basis:

$$\vec{p}(t) = r\hat{e}_r + z(t)\hat{k}$$

$$\vec{v}(t) = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k}$$

$$\vec{a}(t) = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{k}$$

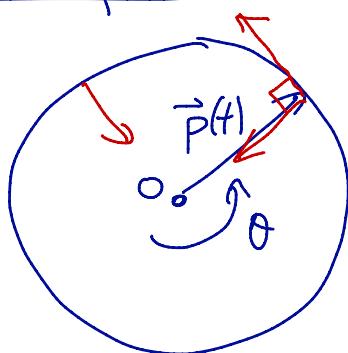
$$\hat{e}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$$

$$\hat{e}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

To make everything easier to visualize assume $z(t) \equiv 0$

example:

$r = \text{const}$, circular motion.



$$\vec{p}(t) = r\cos\theta(t)\hat{i} + r\sin\theta(t)\hat{j}$$

$$= r\hat{e}_r(t)$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \quad \text{speed} = |r\dot{\theta}|$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$= \underbrace{-r\dot{\theta}^2\hat{e}_r}_{\text{radial}} + \underbrace{r\ddot{\theta}\hat{e}_\theta}_{\text{tang.}}$$

$$\text{IF } \omega = \dot{\theta} = \text{const} \iff \vec{a} = -r\dot{\theta}^2\hat{e}_r$$