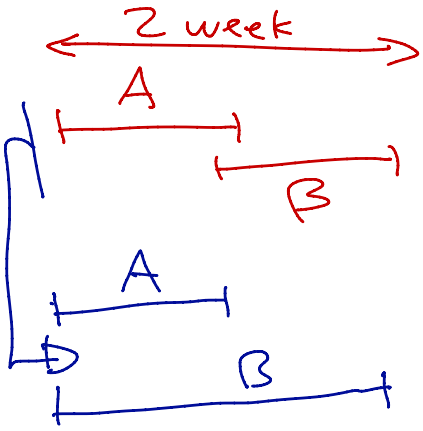
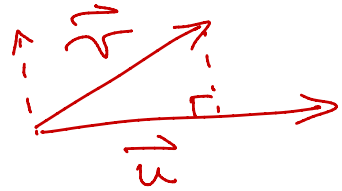
09/11/17

Lecture # 6

Announcement: • iClicker ☑ • first questions: Friday.
• register device



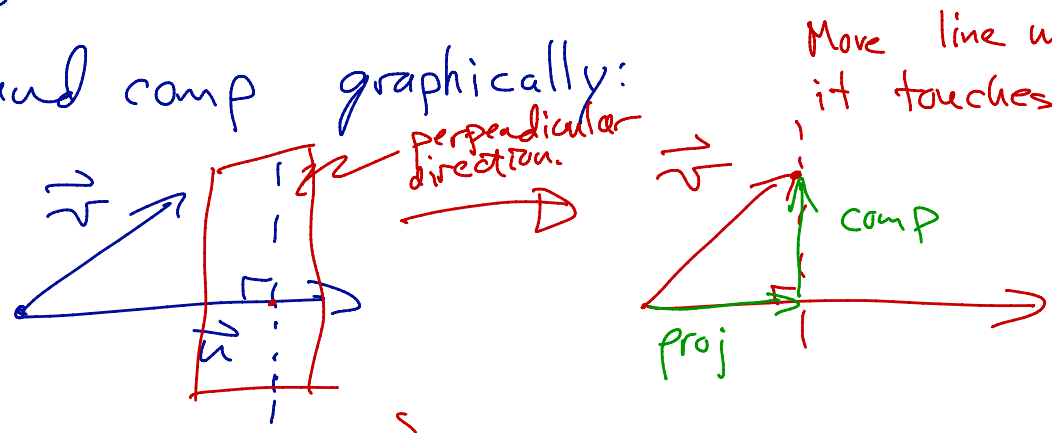
Review: Projection & Complementary Proj



$$\text{proj}(\vec{v}, \vec{u}) = (\vec{v} \cdot \hat{u})\,\hat{u}$$

$$\text{comp}(\vec{v}, \vec{u}) = \vec{v} - \text{proj}(\vec{v}, \vec{u})$$

Finding proj and comp graphically:



$$\vec{v} = \text{proj} + \text{comp}.$$

Q: 

Suppose $\vec{u}$ and $\vec{v}$ are colinear.

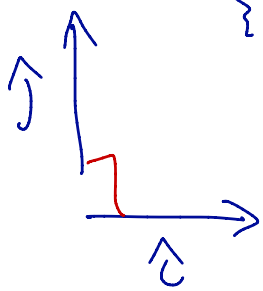
What is $\text{proj}(\vec{v}, \vec{u})$?

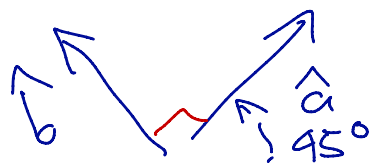
$$\text{proj}(\vec{v}, \vec{u}) = \vec{v} \quad ☑$$

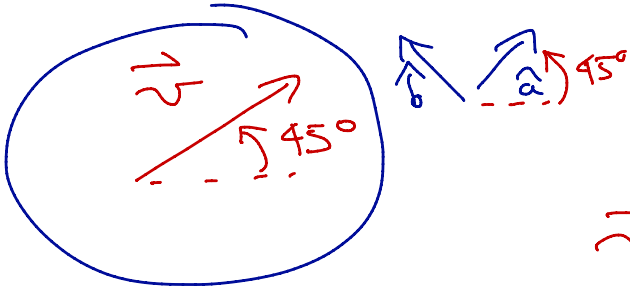
Bases (express vectors in terms of components)

The idea of a basis is to have a standard way of writing and computing with vectors.

$\{\hat{\imath}, \hat{\jmath}\}$ basis







Vectors can be written out in any basis:

$$\vec{v} = \alpha_1 \hat{\imath} + \alpha_2 \hat{\jmath}$$ components are wrt $\hat{\imath}, \hat{\jmath}$

$$= \beta_1 \hat{a} + \beta_2 \hat{b}$$

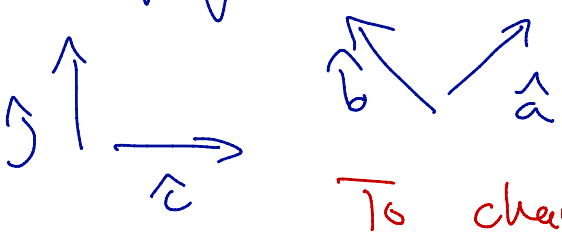
$$\|\vec{v}\| = \sqrt{\alpha_1^2 + \alpha_2^2} = \sqrt{\beta_1^2 + \beta_2^2}$$

$$\| = \sqrt{\vec{v} \cdot \vec{v}}$$

<u>example:</u> $\vec{v} = 2\hat{\imath} + 2\hat{\jmath}$ $\qquad \|\vec{v}\| = 2\sqrt{2}$

$$= 2\sqrt{2}\,\hat{a} + 0 \cdot \hat{b} = \underline{2\sqrt{2}\,\hat{a}}$$

<u>Changing Basis</u>



$$\vec{u} = 3\hat{\imath} + 2\hat{\jmath} \qquad (\star)$$

$$= \underline{\beta_1 \hat{a} + \beta_2 \hat{b}}$$

To change to the $\{\hat{a}, \hat{b}\}$ basis we express $\hat{\imath}$ and $\hat{\jmath}$ in terms of these.

$$\hat{\imath} = \text{proj}(\hat{\imath}, \hat{a}) + \text{proj}(\hat{\imath}, \hat{b}) = \frac{1}{\sqrt{2}}\hat{a} - \frac{1}{\sqrt{2}}\hat{b}$$

$$\hat{\jmath} = \text{proj}(\hat{\jmath}, \hat{a}) + \text{proj}(\hat{\jmath}, \hat{b}) = \frac{1}{\sqrt{2}}\hat{a} + \frac{1}{\sqrt{2}}\hat{b}$$

Substitute into $(\star)$ for $\hat{\imath}$ & $\hat{\jmath}$:

$$\Longrightarrow \quad \vec{u} = 3\left(\frac{1}{\sqrt{2}}\hat{a} - \frac{1}{\sqrt{2}}\hat{b}\right) + 2\left(\frac{1}{\sqrt{2}}\hat{a} + \frac{1}{\sqrt{2}}\hat{b}\right)$$

$$= \frac{5}{\sqrt{2}}\hat{a} - \frac{1}{\sqrt{2}}\hat{b}$$ this is in the new basis $\beta_1 = 5/\sqrt{2}$, $\beta_2 = -\frac{1}{\sqrt{2}}$

<u>Check:</u> $\|\vec{u}\| = \|3\hat{\imath} + 2\hat{\jmath}\| = \sqrt{9+4} = \sqrt{13}$

$$\|\vec{u}\| = \left\| 5/\sqrt{2}\,\hat{a} - \frac{1}{\sqrt{2}}\hat{b} \right\| = \sqrt{\frac{25}{2} + \frac{1}{2}} = \sqrt{13}$$ ✓

Recall: $\vec{v} = 2\hat{\imath} + 2\hat{\jmath} = 2\sqrt{2}\,\hat{a}$

$$\vec{v}\cdot\vec{u} = (2\hat{\imath}+2\hat{\jmath})\cdot(3\hat{\imath}+2\hat{\jmath}) = 6+4 = 10$$

$$\vec{v}\cdot\vec{u} = (2\sqrt{2}\,\hat{a})\cdot\left(\tfrac{5}{\sqrt{2}}\hat{a} - \tfrac{1}{\sqrt{2}}\hat{b}\right) = 10$$

$\boxed{\text{All vector operations are basis independent}}$

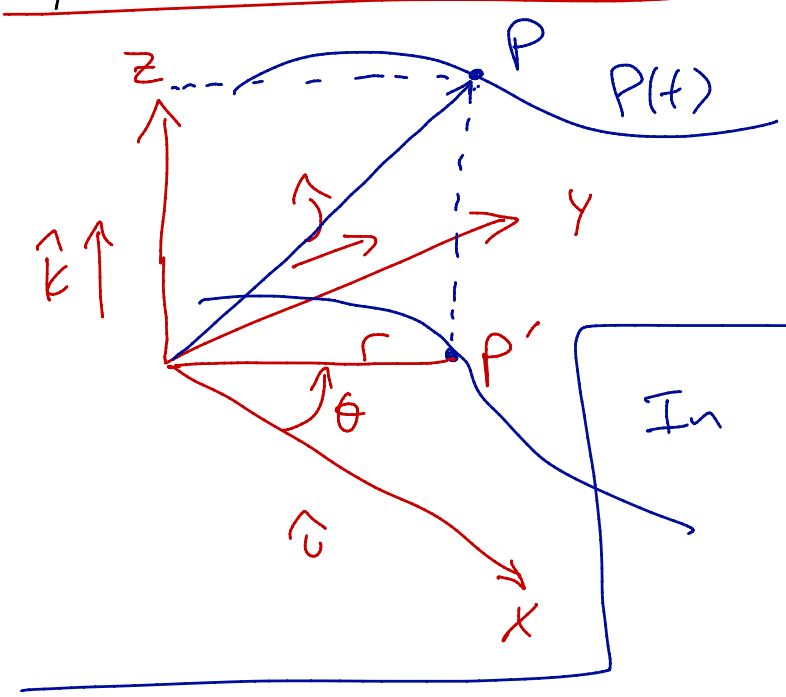
Cylindrical Coordinates

"height above the x-y plane"

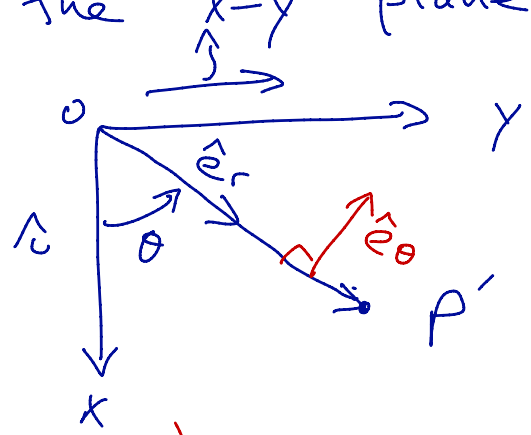
(r, θ, z)

shortest distance from z-axis

the angle with the x-axis in the plane



In the x-y plane



$$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$$\hat{e}_\theta = \hat{k}\times\hat{e}_r = \hat{k}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$= \cos\theta\,\hat{k}\times\hat{\imath} + \sin\theta\,\hat{k}\times\hat{\jmath}$$

$$= -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$\theta(t)$ could be changing.