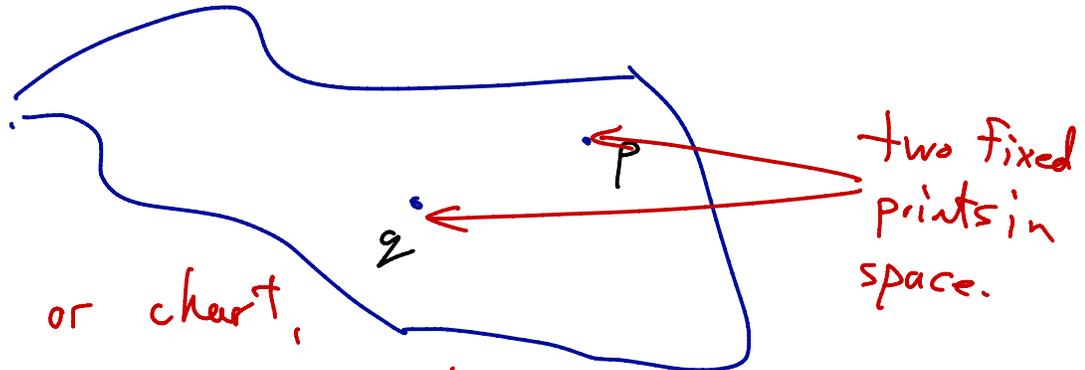


09/06/17

Lecture #4

# Positions, Coordinates and Charts

A position is a point or location in physical space. In TAM212 this means 2D or 3D



Coordinate chart, or chart,

is a function that maps each position in space to a unique set of numbers called coordinates

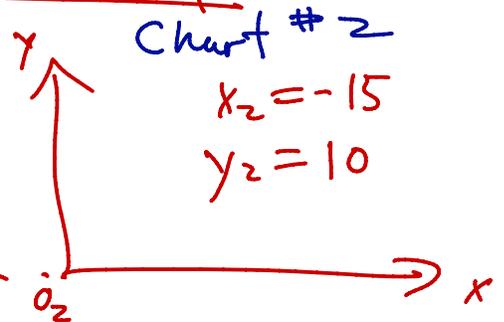
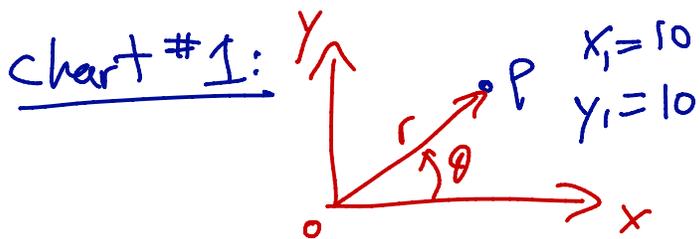
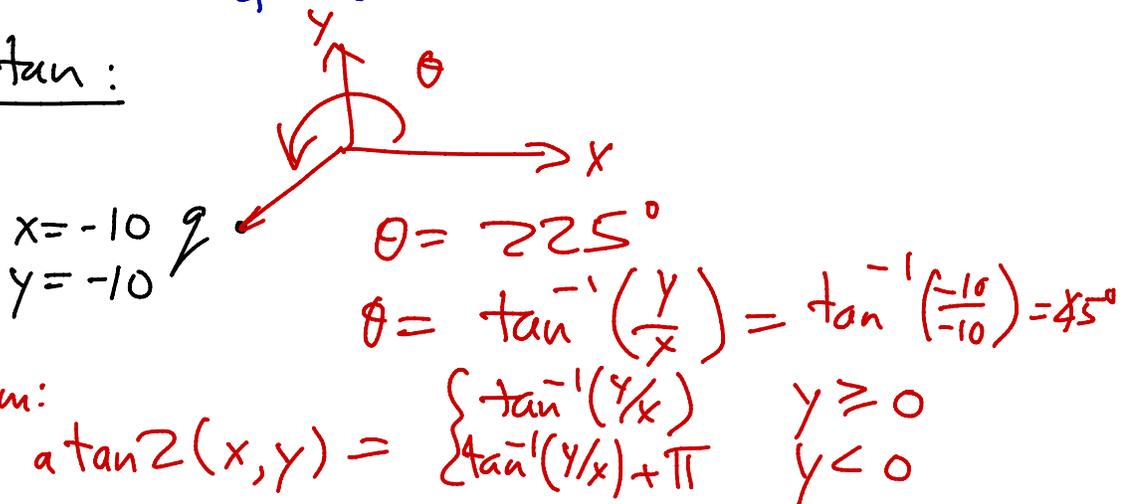


Chart #3 Same axes as Chart #1, but this chart uses polar coords.

$$r = 10\sqrt{2}$$

$$\theta = \frac{\pi}{4} (45^\circ)$$

Aside on arc tan:



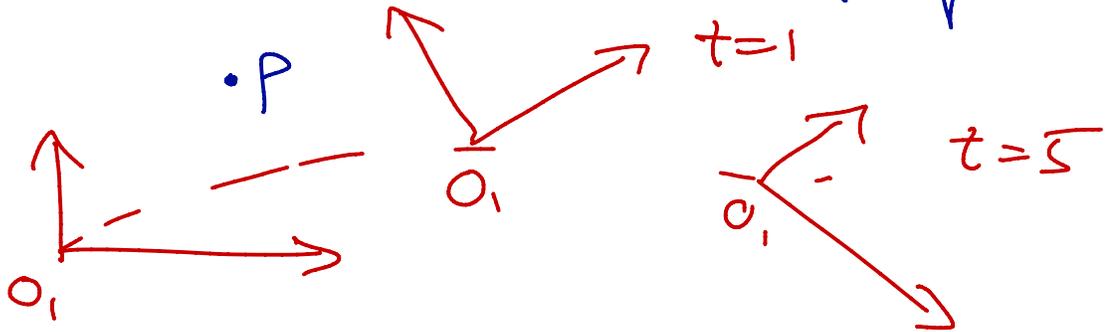
To avoid the problem:

$$a \tan 2(x, y) = \begin{cases} \tan^{-1}(y/x) & y \geq 0 \\ \tan^{-1}(y/x) + \pi & y < 0 \end{cases}$$

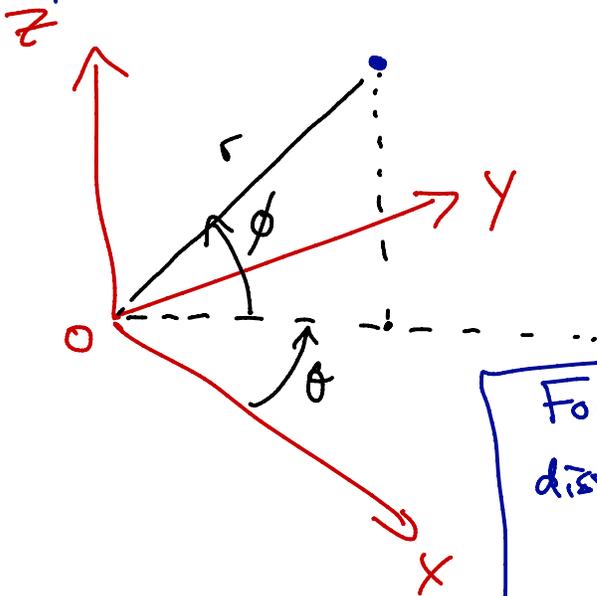
# Coordinates have dimensions and units

dimension	unit	unit
length	meter	feet
time	minute	hours.

If we use charts that change in time, then we need to know the time when computing coords



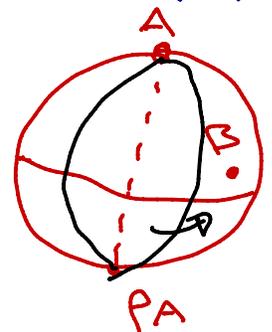
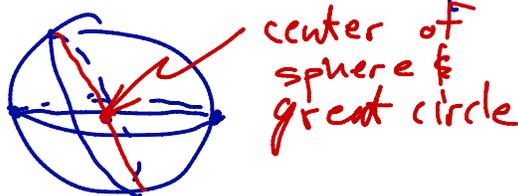
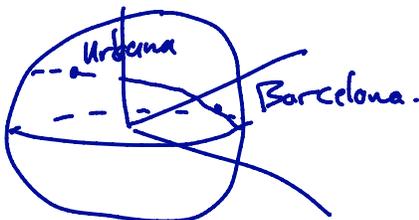
## Spherical coords



- $r$  - radius  $[0, \infty)$
- $\theta$  - azimuth  $[0, 2\pi)$
- $\phi$  - elevation  $[-\pi/2, \pi/2]$

Focus of discussions will be on shortest distances between points on a sphere

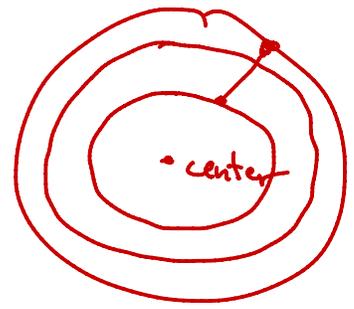
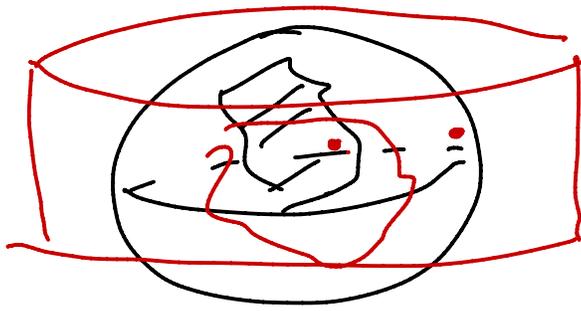
Great circle: any circle on the sphere that has the same radius as the sphere



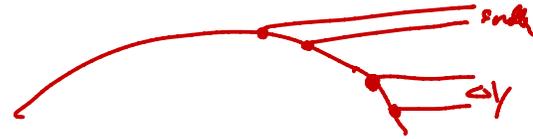
Mercator projection:

maps

top view



side view



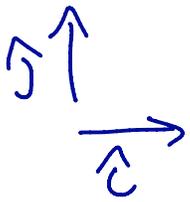
Vectors:

$\vec{v}$  "Directed lengths"

o addition operation

o can multiply by a number (scalar)

Vector components



In 2D any vector  $\vec{v}$  can be written in terms of these two components

$$\vec{v} = x\hat{i} + y\hat{j}$$

coordinates

Unit vector:

a vector whose length is one:

$$\hat{v} = \frac{1}{\|\vec{v}\|} \vec{v}$$

$\|\vec{v}\|$  length of  $\vec{v}$ .

unit vectors are dimensionless.

coordinates have dimensions & units.

# Vector products:

defined on all vectors dot product:  $\vec{v} \cdot \vec{u}$

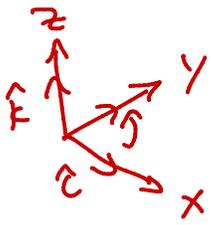
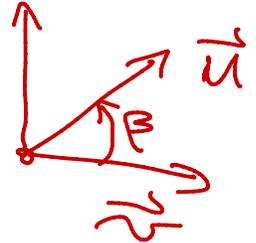
number

3D only  $\rightarrow$  cross product:  $\vec{v} \times \vec{u}$

another vector

## Cross product:

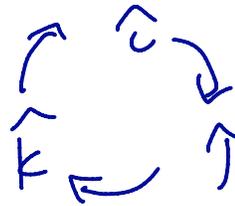
$$\vec{v} \times \vec{u} = (\|\vec{v}\| \|\vec{u}\| \sin \beta) \hat{n}$$



$\hat{i}, \hat{j}, \hat{k}$  orthogonal system

$$\begin{aligned} \hat{i} \times \hat{j} &= \hat{k} \\ \hat{j} \times \hat{k} &= \hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} \end{aligned}$$

$$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$$



Property:  $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

example:  $\hat{j} \times \vec{v} = \hat{j} \times (x\hat{i} + y\hat{j} + z\hat{k})$