

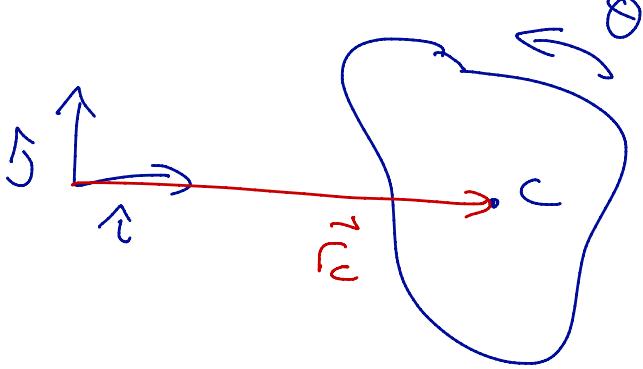
12-06-17

Lecture #35

Last lecture:

kinetic energy of a rigid body is given by

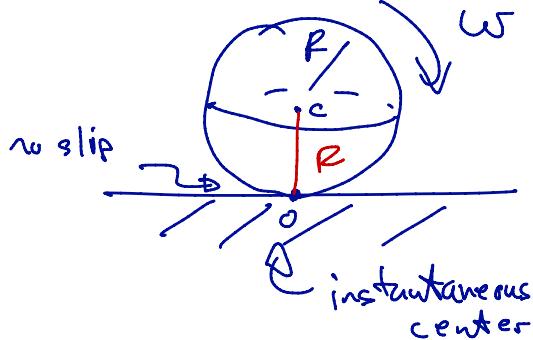
$$\underline{T = \frac{m}{2} \|\dot{\vec{r}_c}\|^2 + \frac{I_c}{2} \dot{\theta}^2}$$



Also if M is the instantaneous center, then

$$\underline{T = \frac{I_M}{2} \dot{\theta}^2}$$

example: rolling cue ball (no slip)



$$\begin{aligned} T_{\text{ball}} &= \frac{I_0}{2} \omega^2 \\ &= \frac{7}{10} mR^2 \omega^2 \\ &= \frac{7}{10} m v_c^2 \end{aligned}$$

$$\text{Alternatively, } T_{\text{ball}} = \frac{m}{2} v_c^2 + \frac{I_c}{2} \omega^2$$

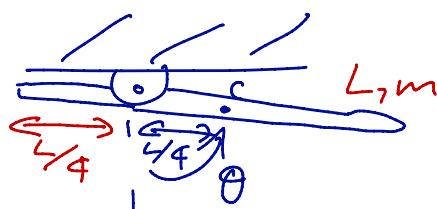
$$\begin{aligned} &= \frac{m}{2} R^2 \omega^2 + \frac{1}{2} \left(\frac{2}{5} mR^2 \omega^2 \right) \\ &= \underline{\frac{7}{10} mR^2 \omega^2} \end{aligned}$$

find: T_{ball}

parallel axis θ^u :

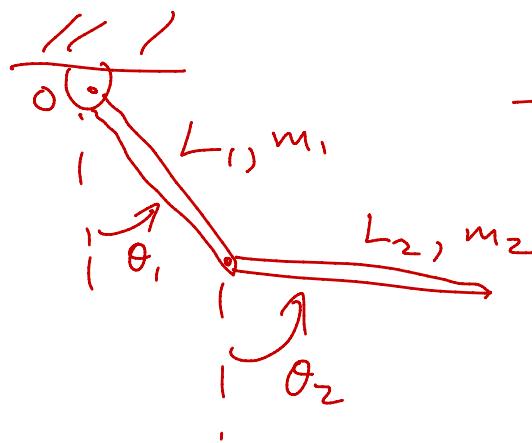
$$\begin{aligned} I_0 &= mR^2 + I_c \\ &= mR^2 + \frac{2}{5} mR^2 \\ &= \underline{\frac{7}{5} mR^2} \end{aligned}$$

example : Kinetic energy of bar pendulum.



$$T = \frac{m}{2} (\vec{v}_c)^2 + \frac{I_c}{2} \omega^2 = \frac{m}{2} \left(\frac{L}{4} \omega \right)^2 + \frac{1}{2} \left(\frac{1}{12} m L^2 \right) \omega^2$$

example :



Find: kinetic energy of this 2-link system

$$T = T_1 + T_2$$

$$T_1 = \frac{I_1}{2} \omega_1^2 = \frac{1}{2} \left(\frac{m}{3} L_1^2 \right) \omega_1^2 = \frac{1}{6} m_1 L_1^2 \omega_1^2$$

Obtaining T_2 :

Need \vec{r}_{c_2} :

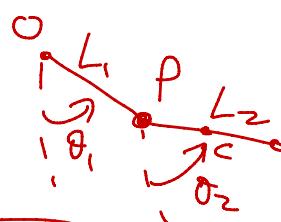
$$\vec{r}_{c_2} = \vec{r}_P + \vec{\omega}_2 \times \vec{r}_{Pc_2}$$

$$\vec{r}_P = \vec{\omega}_1 \times \vec{r}_{OP}$$

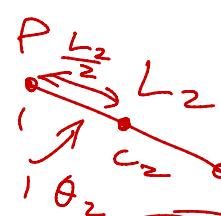
$$\vec{r}_{OP} = L_1 \sin \theta_1 \hat{i} - L_1 \cos \theta_1 \hat{j}$$

$$\vec{r}_P = (L_1 \omega_1) (\cos \theta_1 \hat{i} + \sin \theta_1 \hat{j})$$

$$\vec{\omega}_2 \times \vec{r}_{Pc_2} = \left(\frac{L_2}{2} \omega_2 \right) (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j})$$



$$I_c = \frac{1}{12} m_2 L_2^2$$



$$T_2 = \frac{m}{2} \vec{v}_{c_2}^2 + \frac{I_c}{2} \dot{\theta}_2^2$$

$$\vec{v}_c = \underbrace{(L, \omega, \cos\theta_1 + \frac{\omega_z}{2} \omega_z \cos\theta_z)}_a \hat{e} + \underbrace{(L, \omega, \sin\theta_1 + \frac{\omega_z}{2} \omega_z \sin\theta_z)}_b \hat{g}$$

$$\|\vec{v}_c\|^2 = a^2 + b^2$$