12/04/17

Lecture #34

Announcement!! From one of your fellow 212 students:

Extending the efforts of Domestic Violence Awareness Month,
INNER VOICES Social Issues Theatre presents:

stories that need to be told

Each actor gives voice to a different story, bringing a deeper understanding of the dynamics and impacts of intimate partner violence.

A discussion follows, offering further insight into the complex issues surrounding intimate partner violence.

Channing Murray Foundation
1209 W. Oregon Street, Urbana
Wednesday, November 29, 8:00 p.m.

Armory Free Theatre
room 160 Armory, Champaign
Thursday, November 30, 8:00 p.m.

The Women's Resources Center
616 East Green Street., Suite 202 , Champaign
Wednesday, December 6, 8:00 p.m.

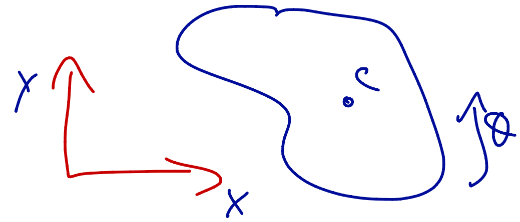
Sherman Hall
909 S. Fifth Street , Champaign
Thursday, December 7, 8:00 p.m.

free and open to the public
contact 217-244-0212 or innervoices@illinois.edu

This event is sponsored in part by The Women's Resources Center
INNER VOICES Social Issues Theatre
is co-sponsored by the Illinois Theatre Department.

ILLINOIS
Counseling Center
counselingcenter.illinois.edu
217-333-3704

Planar Equations of Motion



simplified from 3D.

$$m\ddot{x}_c = F_x^{tot}$$
$$m\ddot{y}_c = F_y^{tot}$$
$$I_{c,\hat{k}}\ddot{\theta} = M_{c,z}^{tot} = \vec{M}^{tot}\cdot\hat{k}$$

In 3D:

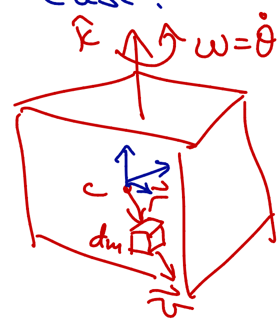
trans: $m\ddot{\vec{r}}_c = \vec{F}^{tot}$

rot: $\dot{\vec{H}}_c = \vec{M}^{tot}$

Derivation of equations:

translational: clear since $\ddot{z}\equiv 0$. $\dot{z}\equiv 0$

rotational: want to show $\dot{\vec{H}}_c = \vec{M}^{tot}$ reduces to $I_{c,\hat{k}}\ddot{\theta} = M_{c,z}^{tot}$ in the planar case.



position vector from center of mass.

angular momentum: $\vec{H}_c = \int \vec{r}\times\vec{v}\,dm$

We know that $\vec{v} = \vec{v}_c + \vec{\omega}\times\vec{r}$

$$\Longrightarrow \vec{H}_c = \int_{body} \vec{r}\times(\vec{v}_c + \vec{\omega}\times\vec{r})\,dm$$

$$\vec{H}_c = \int_{body} \vec{r}\times\vec{v}_c\,dm + \int_{body} \vec{r}\times(\vec{\omega}\times\vec{r})\,dm$$

Our planar motion assumption means that $\vec{\omega} = \omega\hat{k} = \dot{\theta}\hat{k}$

let's write $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$$\vec{\omega}\times\vec{r} = \omega\hat{k}\times(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) = \omega a\hat{\jmath} - \omega b\hat{\imath}$$

$$\vec{r}\times(\vec{\omega}\times\vec{r}) = (a\hat{\imath} + b\hat{\jmath} + c\hat{k})\times(\omega a\hat{\jmath} - \omega b\hat{\imath})$$
$$= \underbrace{\omega a^2\hat{k} + \omega b^2\hat{k}}_{\omega(a^2+b^2)\hat{k}} - \omega ac\,\hat{\imath} - \omega bc\,\hat{\jmath}$$

$$\Rightarrow \vec{H}_c = \int_{body} \omega(a^2+b^2)\,dm\,\hat{k} - \left(\int_{body} \omega ac\right)\hat{i} - \left(\int \omega bc\,dm\right)\hat{j}$$

We know that $\dot{\vec{H}}_c = \vec{M}^{net}$ moments about $\hat{i}, \hat{j}, \hat{k}$ axes.

$$\Rightarrow \boxed{I_{zz}\,\dot{\omega} = \vec{M}^{net} \cdot \hat{k}}$$

$$I_{zz}\,\ddot{\theta} = M^{net}_{c,\hat{k}}$$

Principle of Work and Energy

$$W = \Delta T$$

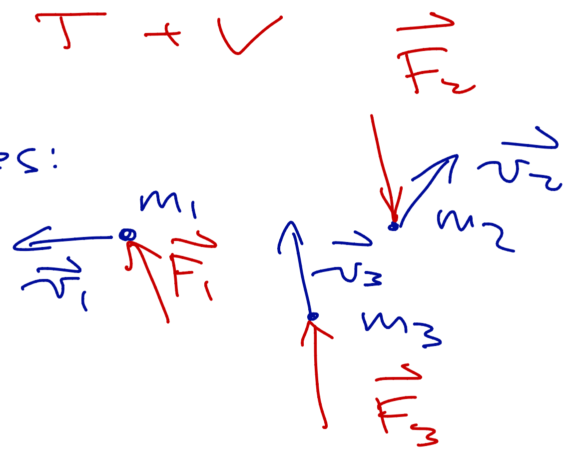
work done = change in K.E.

change in kinetic energy T.

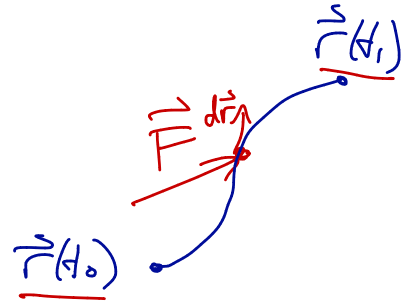
Total energy = kinetic + potential

$$E = T + V$$

System of particles:



Kinetic energy of single part:



$$T = \frac{m}{2}\|\vec{v}\|^2$$

Let's relate to work:

$$\frac{dT}{dt} = \frac{d}{dt}\left\{\frac{m}{2}\vec{v}\cdot\vec{v}\right\} = \frac{m}{2}\left\{\dot{\vec{v}}\cdot\vec{v} + \vec{v}\cdot\dot{\vec{v}}\right\}$$

$$= (m\dot{\vec{v}})\cdot\vec{v} = \vec{F}\cdot\vec{v}$$ or rate of work or power.

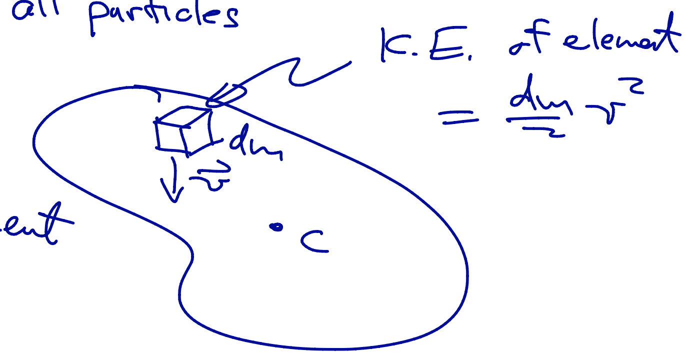
$$\int_{t_0}^{t_1}\frac{dT}{dt}dt = \int_{t_0}^{t_1}\vec{F}\cdot\vec{v}\,dt \qquad \vec{v}\,dt = d\vec{r} \qquad \vec{v} = \frac{d\vec{r}}{dt}$$

$$T(t_1) - T(t_0) = \int_{\text{path}}\vec{F}\cdot d\vec{r}$$

recognize as the work done on the particle moving along the path from $r(t_0)$ to $r(t_1)$.

$$\text{System kinetic energy} = \sum_{\text{all particles}}\frac{m_i}{2}v_i^2$$

Rigid Body Case



As a limit letting the element size $\rightarrow 0$:

$$T = \frac{1}{2}\int_{\text{body}}\|\vec{v}\|^2 dm \qquad \|\vec{a}\|^2 = \vec{a}\cdot\vec{a}$$

$$= \frac{1}{2}\int_{\text{body}}\overbrace{\|\vec{v}_c + \vec{\omega}\times\vec{r}\|^2}\,dm$$

$$T = \frac{1}{2}\int \|\vec{v}_c + \vec{\omega}\times\vec{r}\|^2 \, dm$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \|\vec{a}\|^2 + \underbrace{\vec{b}\cdot\vec{a} + \vec{a}\cdot\vec{b}}_{2\vec{a}\cdot\vec{b}} + \|\vec{b}\|^2$$

$$\{\vec{c}\cdot(\vec{a}\times\vec{b}) = (\vec{c}\times\vec{a})\cdot\vec{b}\}$$

$$= \frac{1}{2}\left(\int \|\vec{v}_c\|^2 dm + \int \underbrace{\vec{v}_c\cdot(\vec{\omega}\times\vec{r})}_{(\vec{v}_c\times\vec{\omega})\cdot\vec{r}}\, dm \to 0\right.$$

$$\left. + \frac{1}{2}\int \|\vec{\omega}\times\vec{r}\|\, dm\right)$$

$$\int \vec{v}_c\cdot(\vec{\omega}\times r)\, dm = \int (\vec{v}_c\times\vec{\omega})\cdot\vec{r}\, dm = (v_c\times\vec{\omega})\cdot\cancel{\int \vec{r}\, dm} \to 0$$

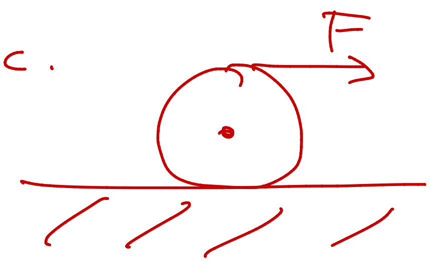
$$= \frac{m}{2}\|\vec{v}_c\|^2 + \frac{1}{2}\int \|\omega\hat{k}\times(a\hat{i}+b\hat{j})\|^2 dm$$

$$= \frac{m}{2}\|\vec{v}_c\|^2 + \frac{\omega^2}{2}\int \underbrace{\|a\hat{j} - b\hat{i}\|^2}_{a^2+b^2} dm$$

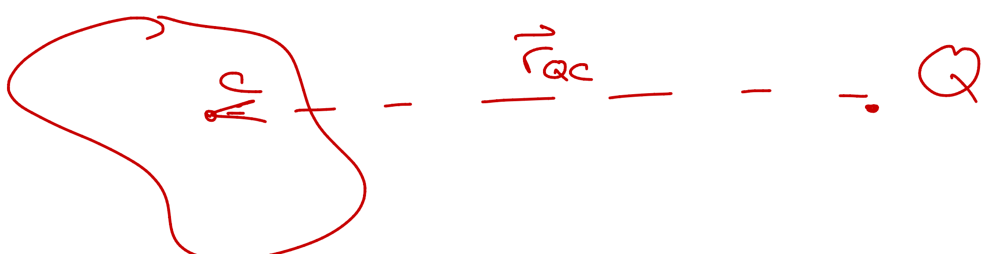
$$T = \frac{m}{2}\|\vec{v}_c\|^2 + \frac{I_c}{2}\omega^2$$

trans. kinetic

rot. kinetic.



We can also express T wrt the instantaneous cent of zero velocity Q.

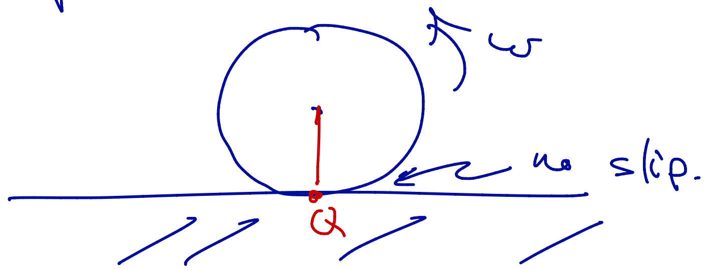


$$T = \frac{m}{2}v_c^2 + \frac{I_c}{2}\omega^2 \qquad |v_c|^2 = |r_{QC}\,\omega|^2$$

$$= \frac{m}{2}|r_{QC}\,\omega|^2 + \frac{I_C}{2}\omega^2 = \frac{1}{2}(m r_{QC}^2 + I_C)\,\omega^2$$

$$= \frac{I_{Q,\hat{k}}}{2}\omega^2$$

example: rolling cylinder without slipping.



$$|\omega| = \left|\frac{v_c}{R}\right|$$

$$T = \frac{m}{2} v_c^2 + \frac{I_c}{2} \omega^2$$

$$= \underbrace{\frac{m}{2} v_c^2}_{\frac{2}{3}\text{ trans}} + \underbrace{\frac{1}{2}\left(\frac{m}{2}\cancel{R^2}\right)\left(\frac{v_c}{\cancel{R}}\right)^2}_{\frac{1}{3}\text{ rot}} = \frac{3}{4} m v_c^2$$