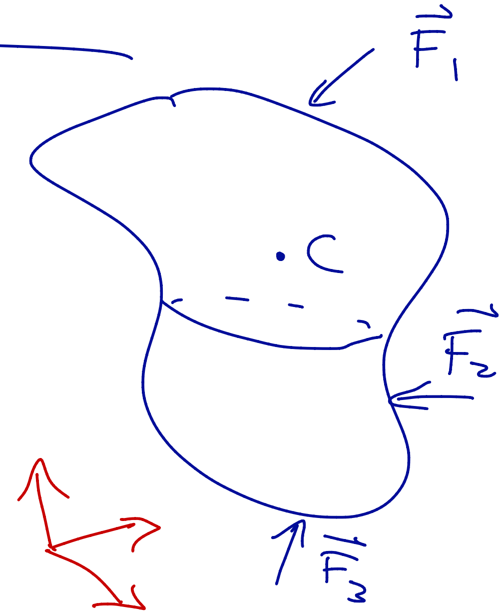
12/01/17

Lecture #33

Motions in the Plane

Our equations of motion for a 3D rigid body are:



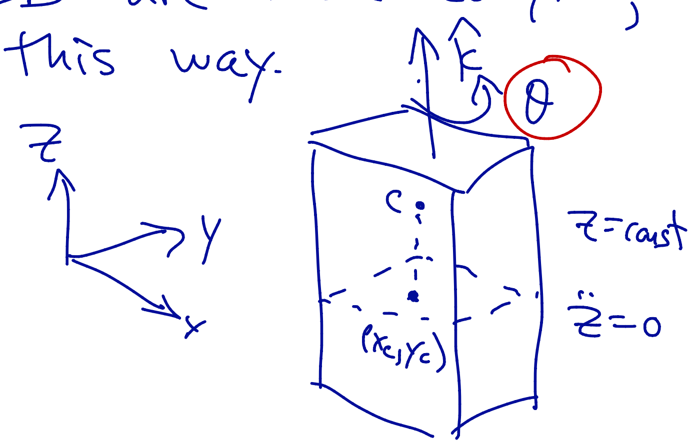
translational: $\dot{\vec{P}}_c = m\ddot{\vec{r}}_c = \vec{F}^{net}$

rotational: $\dot{\vec{H}}_c = \vec{M}_c^{net}$

translational:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \quad \begin{cases} m\ddot{x}_c = F_x^{net} \\ m\ddot{y}_c = F_y^{net} \\ \cancel{m\ddot{z}_c = F_z^{net}} \end{cases}$$

The rotational dynamics in 3D are more complex, and are not decoupled in this way.



Planar case:

EoM become:

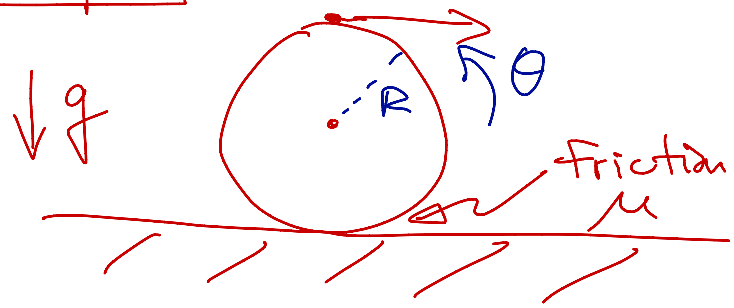
trans: $m\ddot{x}_c = F_x^{net}$

$m\ddot{y}_c = F_y^{net}$

rot: $I_{zz}\ddot{\theta} = M_{c,z}^{net} = \{ \vec{M}_c^{net} \cdot \hat{k} \}$

θ is the rotation about $\hat{k}$-axis example:

$\vec{F}$ = horizontal force

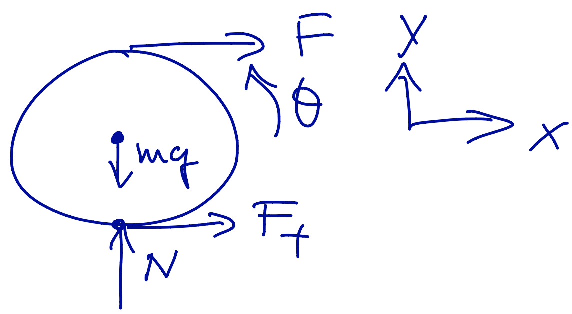


Given: a sphere rolling on a flat surface with friction coeff μ

(I) Find: maximum allowable force F if there is no slipping.

Solution: $I_c^{sphere} = \frac{2}{5} m R^2$

FBD:



gravity acts as though it's a single force at c of m.

x: $m\ddot{x} = F + F_f$

y: $m\ddot{y} = 0$ ⟵ no vertical motion

$= N - mg \Rightarrow N = mg.$

θ: $I_c \ddot{\theta}$ = total moment around c $= (F_f - F)R$

$\Rightarrow \frac{2}{5} m R^2 \ddot{\theta} = (F_f - F)R$

Since we are in a no slip scenario:

$R\theta = -x + \text{const}$

$R\ddot{\theta} = -\ddot{x}$

$-\frac{2}{5} m \cancel{R} \ddot{x} = (F_f - F)\cancel{R}$

$\Rightarrow F + F_f = \frac{5}{2}(F - F_f) \Rightarrow \boxed{F_f = \frac{3}{7} F}$

$$\vec{F}_t = \frac{3}{7} F$$

At max force; i.e., any bigger force would cause slip.

$$|F_t| = \mu |N| = \underline{\mu m g}$$

Substituting we get $\frac{3}{7} F_{max} = \mu m g$

$$\Longrightarrow F_{max} = \underline{\frac{7}{3} \mu m g}$$

(II) Suppose we apply double the max force.

<u>Find:</u> $\ddot{x}$ & $\ddot{\theta}$

<u>solution:</u>

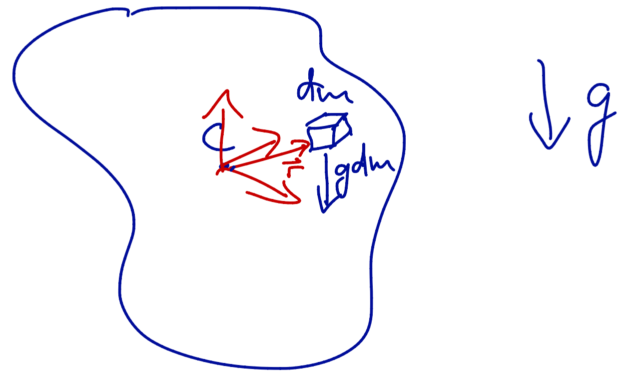
$$F_{max} = \frac{14}{3} \mu m g$$

$F_t = \mu m g$ or it's ~~m~~ extremized.

$$\ddot{x} = \frac{F + F_t}{m} =$$

$$\ddot{\theta} = \left(\frac{1}{I_c^{sphere}} \right) (\mu m g - F) R$$

Check: Rigid body in gravity



total force:

$$\vec{F}^{net} = \int_{body} -\hat{j} g\, dm = -\hat{j} g m$$

total moment about c of m:

$$\vec{M}_c^{net} = \int_{body} \vec{r} \times (-\hat{j} g\, dm) = \left(\int_{body} \vec{r}\, dm \right) \times (-\hat{j} g)$$

$$= \vec{0} \times (-\hat{j} g) \qquad \text{no moment!}$$

Effect of gravity on EoM is the same as letting ~~the~~ a single total force act at the c of m example: