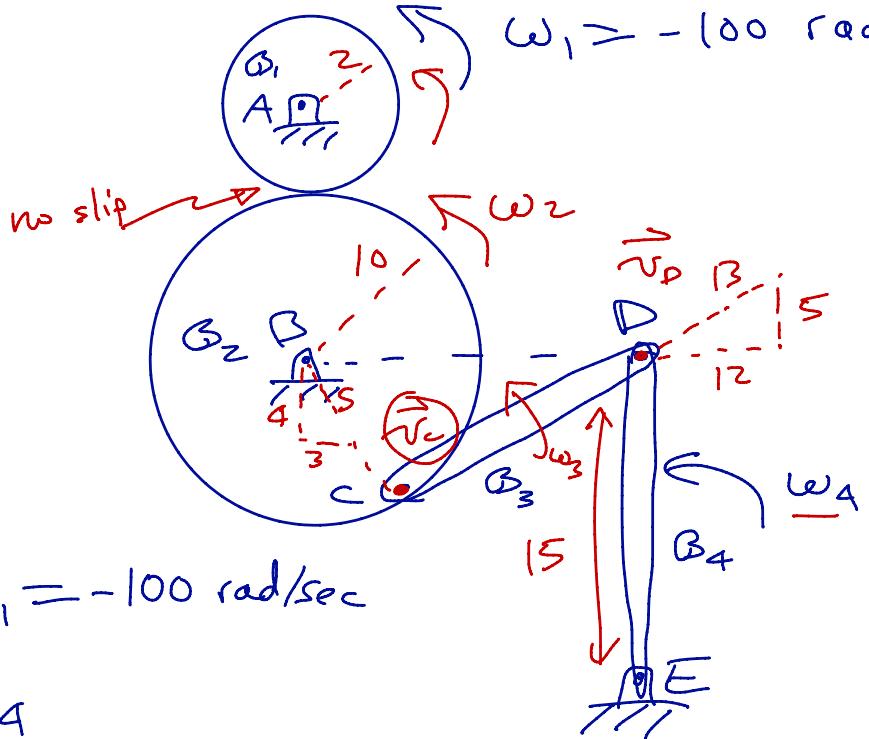


Lecture #30B

example:



Given: $\omega_1 = -100 \text{ rad/sec}$

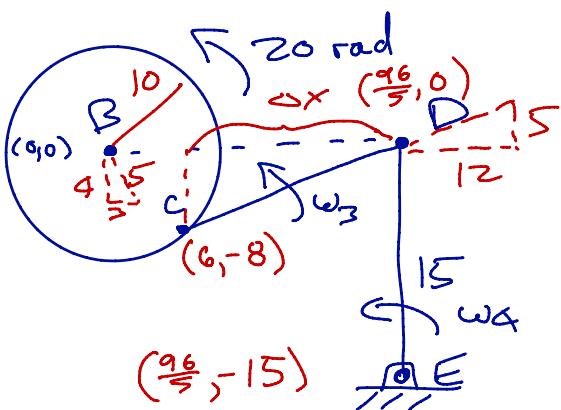
Find: ω_4

Solution: no slip situation and the discs are pinned to rotate about their centers
 \Rightarrow standard gear equation applies.

$$r_1 \omega_1 = -r_2 \omega_2$$

$$-\left(\frac{r_1}{r_2}\right) \omega_1 = \omega_2$$

$$\underline{\underline{20 \text{ rad/sec}}} = -\left(\frac{2}{10}\right)(-100) = \omega_2$$



$$\Delta x = \left(\frac{96}{5}\right)12 = \frac{96}{5}$$

$$\begin{aligned} \vec{v}_c &= \vec{\omega}_2 \times \vec{r}_{Bc} = (20\hat{k}) \times (42\hat{i} - 8\hat{j}) \\ &= 120\hat{j} + 160\hat{k} \end{aligned}$$

$$\vec{v}_D = \vec{v}_c + \vec{\omega}_3 \times \vec{r}_{CD} = \vec{\omega}_4 \times \vec{r}_{ED}$$

$$120\hat{j} + 160\hat{k} + (\omega_3 \hat{k}) \times \left(\frac{96}{5}\hat{i} + 8\hat{j}\right) = (\omega_4 \hat{k}) \times (15\hat{j})$$

$$120\hat{j} + 160\hat{k} + \omega_3 \left(\frac{96}{5}\right)\hat{j} - \omega_3 8\hat{k} = -\omega_4 15\hat{k}$$

$$\cancel{120\downarrow} + \cancel{160\uparrow} + \omega_3 \left(\frac{96}{5}\right)\uparrow - \cancel{\omega_3 8\uparrow} = -\omega_4 15 \uparrow$$

$\Rightarrow \uparrow: 120 + \omega_3 \left(\frac{96}{5}\right) = 0$

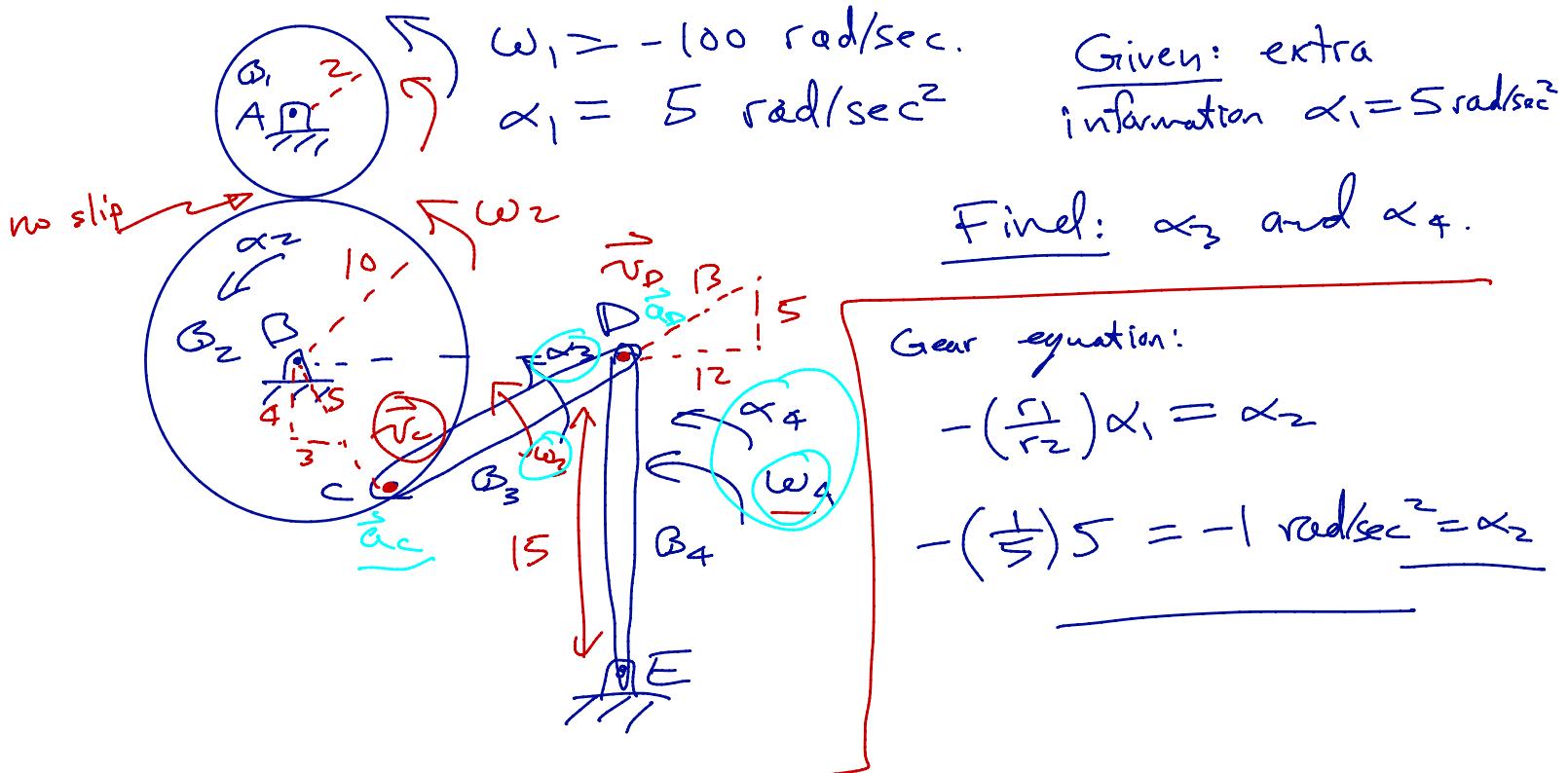
$$\omega_3 = -120 \left(\frac{5}{96}\right) = -10 \left(\frac{5}{8}\right)$$
$$\omega_3 = \underline{-50/8}$$

C: $160 - \left(-\frac{50}{8}\right)8 = -\omega_4 15$

$$-\frac{210}{15} = \omega_4$$

✓

example cont'd:



Gear equation:

$$-(\frac{r_1}{r_2})\alpha_1 = \alpha_2$$

$$-(\frac{1}{5})5 = -1 \text{ rad/sec}^2 = \underline{\alpha_2}$$

$$\vec{a}_c = \vec{\alpha}_2 \times \vec{r}_{BC} - \omega_2^2 \vec{r}_{BC}$$

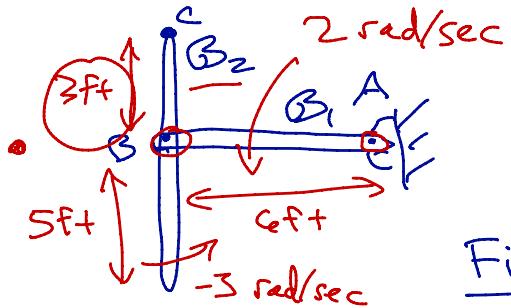
$$\vec{a}_D = \vec{a}_c + \vec{\alpha}_3 \times \vec{r}_{CD} - \omega_3^2 \vec{r}_{CD} = \vec{\alpha}_4 \times \vec{r}_{ED} - \omega_4^2 \vec{r}_{ED}$$

$$\vec{a}_c = (-\vec{k}) \times (6\hat{i} - 8\hat{j}) - (20)^2 (6\hat{i} - 8\hat{j})$$

$$\begin{aligned} \vec{a}_c &+ (\cancel{\vec{\alpha}_3} \hat{k}) \times \left(\frac{96}{5} \hat{i} + 8\hat{j} \right) - \left(\frac{25}{4} \right)^2 \left(\frac{96}{5} \hat{i} + 8\hat{j} \right) \\ &= (\cancel{\vec{\alpha}_3} \hat{k}) \times (15\hat{j}) - \left(\frac{250}{16} \right)^2 (15\hat{j}) \end{aligned}$$

Solve the equations for α_3 and α_4 .

example:



Given: $\omega_1 = 2 \text{ rad/sec}$
 $\omega_2 = -3 \text{ rad/sec}$

Solution:

$$\begin{aligned}\vec{v}_B &= \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{AB} \\ &= (2\hat{k}) \times (-6\hat{i}) = -12\hat{j}\end{aligned}$$

The instantaneous center is defined by: $\vec{\omega}_2 \times \vec{r}_{IB} = \vec{v}_B$

$$(-3\hat{k}) \times (a\hat{i} + b\hat{j}) = -12\hat{j}$$

$$-3a\hat{j} + 3b\hat{i} = -12\hat{j} + 0\cdot\hat{i}$$

$$\vec{r}_{IC} = \vec{r}_{IB} + \vec{r}_{BC} = 4\hat{i} + 3\hat{j}$$

$$\begin{aligned}b &= 0; a = 4 \\ \vec{r}_{IB} &= 4\hat{i} \quad \checkmark \\ \vec{r}_{IC} &= \end{aligned}$$

$$\vec{v}_C = \vec{\omega}_2 \times \vec{r}_{IC} = (-3\hat{k}) \times (4\hat{i} + 3\hat{j}) = -12\hat{j} + 9\hat{i} \text{ ft/sec.} \quad \checkmark$$

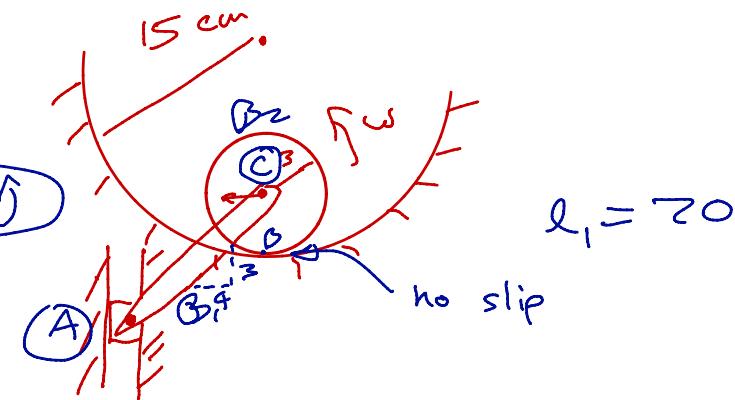
Verify: $\vec{v}_C = \vec{v}_B + \vec{\omega}_2 \times \vec{r}_{BC}$

$$\begin{aligned}&= -12\hat{j} + (-3\hat{k}) \times (3\hat{j}) \\ &= -12\hat{j} + 9\hat{i} \quad \checkmark\end{aligned}$$

example:

Given: $\vec{v}_A = 12 \text{ cm/sec} \uparrow$

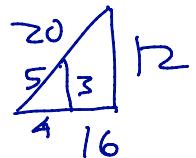
$\vec{a}_A = -6 \text{ cm/sec}^2 \uparrow$



Find: α_1

Solution: will need some angular velocities

$$\vec{\omega}_c = \vec{\omega}_2 \times \vec{r}_{BC} = \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{Ac}$$



$$\vec{v}_E = 9\hat{i} \quad (\vec{\omega}_2 \hat{k}) \times (3\hat{j}) = 12\hat{j} + (\vec{\omega}_1 \hat{k}) \times (16\hat{i} + 12\hat{j})$$

$$-3\omega_2 \hat{c} = 12\hat{j} + 16\omega_1 \hat{j} - 12\omega_1 \hat{i}$$

$$\Rightarrow \hat{c}: 0 = 12 + 16\omega_1 \Rightarrow \boxed{\omega_1 = -3/4}$$

$$-3\omega_2 = -12(-3/4) = 9 \Rightarrow \boxed{\omega_2 = -3}$$

acceleration:

$$\vec{a}_c = \vec{a}_A + \vec{\alpha}_1 \times \vec{r}_{Ac} - \vec{\omega}_1^2 \vec{r}_{Ac} = r\vec{\alpha}_2 \hat{e}_T + \left(\frac{(r\omega_2)^2}{l - r} \right) \hat{e}_n$$

2 eqns 2 scalar unknowns. $\hat{e}_T = -\hat{c}$ $\hat{e}_n = \hat{j}$

Solve to get: $\boxed{\alpha_1 = 1.22 \text{ rad/sec}^2}$ $\rho = 15$