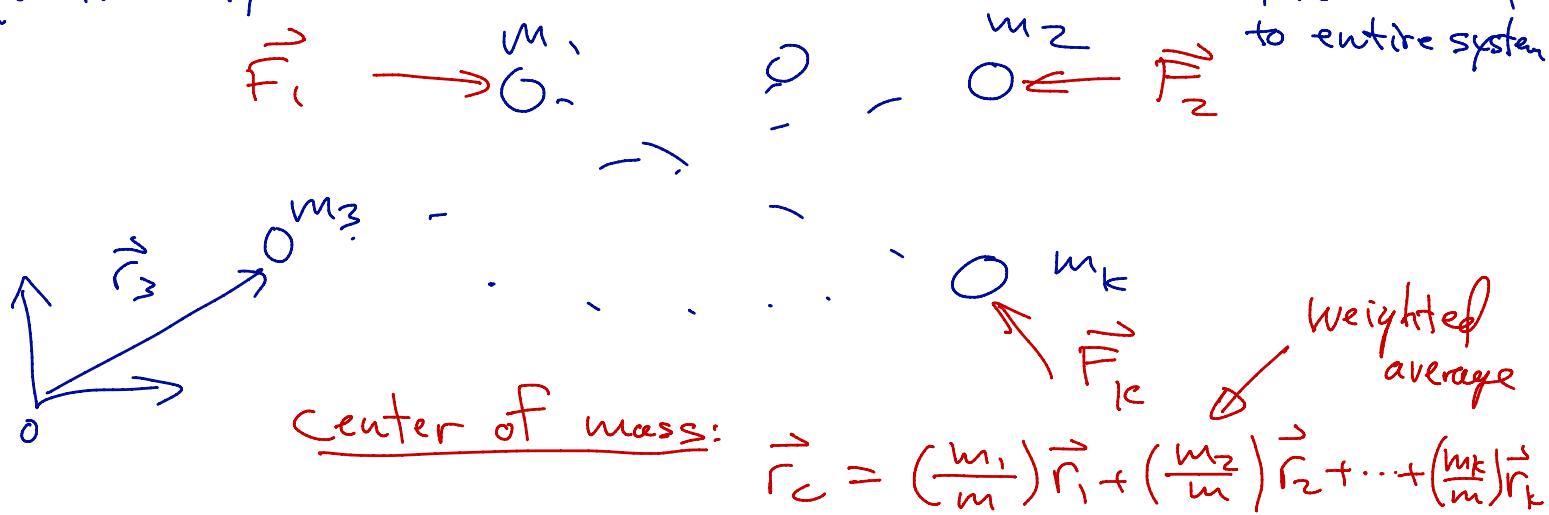


11/10/17

Lecture #28

Kinetics of Particle Systems & Rigid Bodies

particle system



Euler's 1st Law: $m\ddot{\vec{r}}_c = \vec{F}_{\text{tot}}$ \Leftrightarrow total mass $m = m_1 + \dots + m_k$ \Leftrightarrow total of external forces applied to system

example:



The external forces $= \vec{0} = \vec{F}_1 = \vec{F}_2$

$$\Rightarrow m\ddot{\vec{r}}_c = \vec{0} \quad \text{Euler's 1st law}$$

$$\ddot{\vec{r}}_c = \vec{0}$$

$$\vec{r}_c = \vec{v}_c(0) = \left(\frac{m_1}{m_1+m_2}\right)\vec{v}_1(0) + \left(\frac{m_2}{m_1+m_2}\right)\vec{v}_2(0)$$

$$\vec{r}_c(t) = \vec{r}_c(0) + \vec{v}_c(0)t$$

Forces are nonzero:

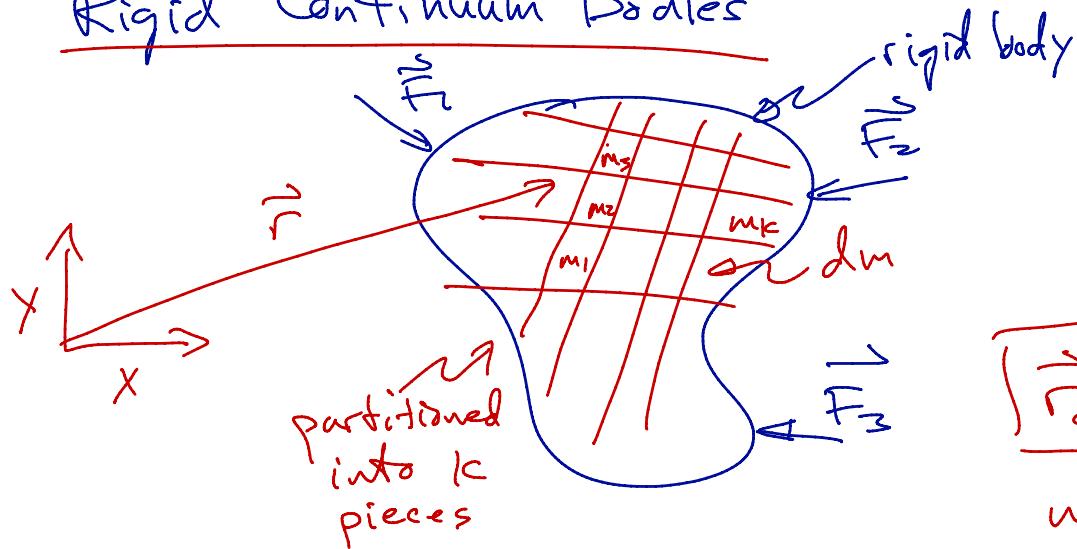
$$m\ddot{\vec{r}}_c = \vec{F}_{\text{tot}} \Rightarrow$$

$$\vec{F}_{\text{tot}} = \vec{F}_1 + \vec{F}_2 = \text{const.}$$

$$\vec{r}_c(t) = \vec{r}_c(0) + \frac{t}{m}\vec{F}_{\text{tot}}$$

$$\vec{r}_c(t) = \vec{r}_c(0) + \vec{v}_c(0)t + \frac{t^2}{2m}\vec{F}_{\text{tot}}$$

Rigid Continuum Bodies

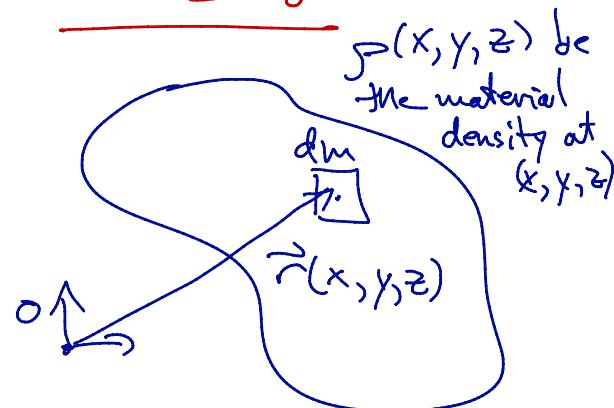


Mass is distributed:
cannot use Newton directly.

$$\boxed{\vec{r}_c \approx \frac{1}{m} \sum m_i \vec{r}_i}$$

$$m = m_1 + \dots + m_k \\ = \text{total mass of rigid body}$$

$$m = \sum m_i$$



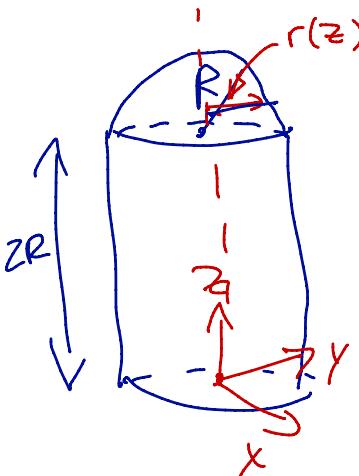
$$\Rightarrow dm = \rho(x, y, z) dV$$

In particular: $\rho = \text{const}$

$$\boxed{m = \rho V_{\text{tot}}} \\ \boxed{\vec{r}_c = \frac{1}{V_{\text{tot}}} \int_{\text{body}} \vec{r} dV}$$

$$\boxed{m \ddot{\vec{r}}_c = \vec{F}_{\text{tot}}}$$

example: rounded cylinder; assume uniform density.



$$\vec{r}_c = \frac{1}{V_{\text{tot}}} \int_{\text{body}} \vec{r} dV$$

$$\vec{r}(x, y, z) = x \hat{i} + y \hat{j} + z \hat{k}$$

$$\vec{r}_c = \frac{1}{V_{\text{tot}}} \left\{ \int_{\text{body}} x dV \hat{i} + \int_{\text{body}} y dV \hat{j} + \int_{\text{body}} z dV \hat{k} \right\}$$

$\int_{\text{body}} x dV$ $\int_{\text{body}} x dV$
 x neg x pos

The computation boils down to:

$$\vec{r}_c = \frac{1}{V_{\text{tot}}} \int_{\text{body}} z dV \hat{k}$$

cylinder

$$\begin{aligned} \vec{r}_{cz} &= \frac{1}{V_{\text{tot}}} \left\{ \int_0^{2R} z (\pi R^2) dz + \int_{2R}^{3R} z (\pi r(z)^2) dz \right\} \\ &= \frac{1}{V_{\text{tot}}} \left\{ \int_0^{2R} \pi R^2 z dz + \int_{2R}^{3R} z (R^2 - (z-2R)^2) \pi dz \right\} \end{aligned}$$

hemisphere:

$$(z-ZR)^2 + x^2 + y^2 = R^2$$

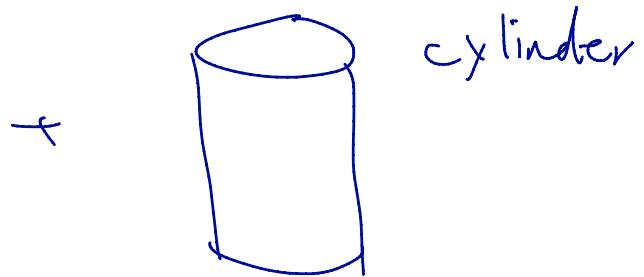
$$x^2 + y^2 = R^2 - (z-ZR)^2 = r(z)^2$$

$$= 1.34 R$$

✓

Second method: using table

Breaks into 2 parts.



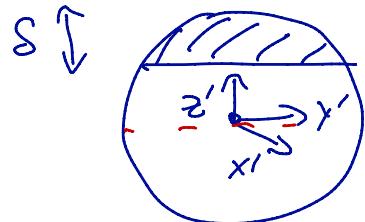
$$\vec{r}_c = \left(\frac{m_{\text{hem}}}{m_{\text{tot}}} \right) \vec{r}_{c_{\text{hem}}} + \left(\frac{m_{\text{cyl}}}{m_{\text{tot}}} \right) \vec{r}_{c_{\text{cyl}}}$$

uniform ρ .

$$= \left(\frac{V_{\text{hem}}}{V_{\text{tot}}} \right) \vec{r}_{c_{\text{hem}}} + \left(\frac{V_{\text{cyl}}}{V_{\text{tot}}} \right) \vec{r}_{c_{\text{cyl}}}$$

If we have $\vec{r}_{c_{\text{hem}}}$ and $\vec{r}_{c_{\text{cyl}}}$ we can immediately compute \vec{r}_c .

$\vec{r}_{c_{\text{hem}}}$



Standard table gives

$$\vec{r}'_{c_{\text{hem}}} = \frac{3}{4} \left(\frac{2R-S}{3R-S} \right) \hat{k}$$

In our case $S=R$:

$$\vec{r}'_{c_{\text{hem}}} = \frac{3}{8} R \hat{k}$$

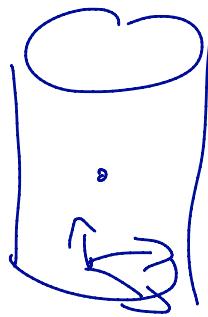
$$\vec{r}_{c_{\text{hem}}} = (2R + \frac{3}{8}R) \hat{k} = \frac{19}{8} R \hat{k}$$

$$V_{\text{hem}} = \frac{2}{3} \pi R^3$$

II Cylinder:

$$\vec{r}_{cyl} = R \hat{k}$$

$$V_{cyl} = 2\pi R^3$$



Putting it together

$$\begin{aligned}\vec{r}_c &= \left(\frac{1}{V_{cyt} + V_{hem}} \right) \left(V_{cyt} \vec{r}_{cyl} + V_{hem} \vec{r}_{hem} \right) \\ &= \underline{1.34R \hat{k}}\end{aligned}$$