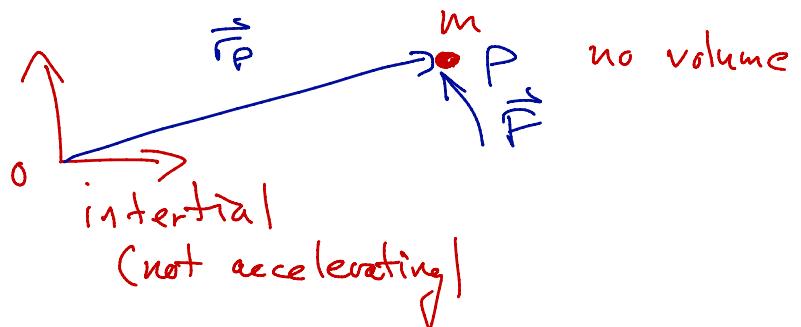


11/08/17

Lecture #27

Kinetics of Particle Systems and Rigid Bodies

Newton's laws: given for point masses



1st and 2nd law:

$$\vec{F} = m\vec{a}$$

3rd law:

Point mass A

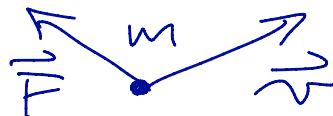


Point mass B



When Particle A exerts a force \vec{F}_{AB} on Particle B
then Particle B exerts $\vec{F}_{BA} = -\vec{F}_{AB}$ on Particle A.

Linear momentum: point mass



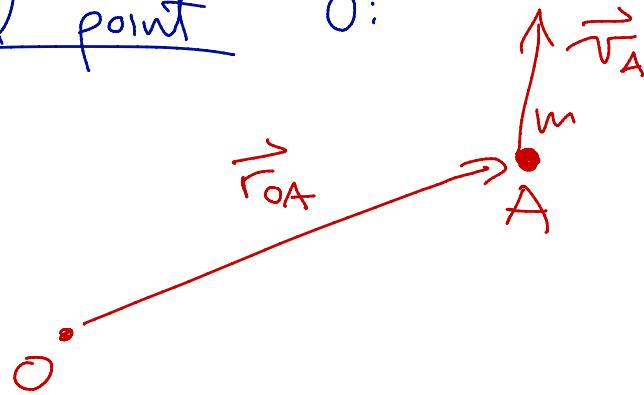
Define linear momentum as $\vec{p} = m\vec{v}$

$$\vec{F} = m\vec{a} = m \frac{d}{dt} \{ \vec{v} \} = \frac{d\vec{p}}{dt}$$

$\boxed{\vec{p} = \vec{F}}$

Angular Momentum: point mass

Given a fixed point O:

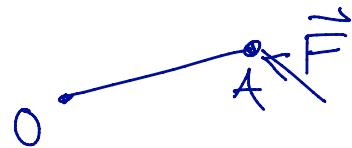


We define the angular momentum of the point mass about point O as

$$\vec{H}_o = \vec{r}_{OA} \times \vec{p} \quad \{ = \vec{r}_{OA} \times (m \vec{v}_A) \}$$

Moments: generated by force \vec{F} about point O.

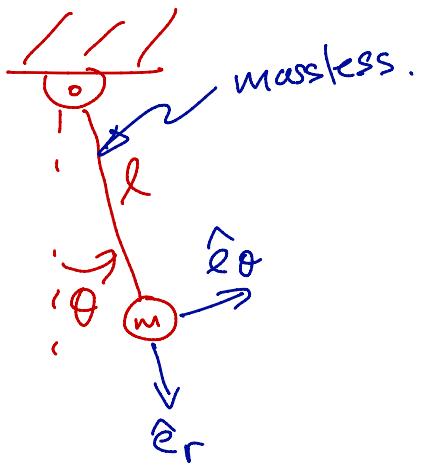
$$\vec{M}_o = \vec{r}_{OA} \times \vec{F}$$



Moment equation:

$$\begin{aligned} \frac{d}{dt} \{ \vec{H}_o \} &= \frac{d}{dt} \{ (\vec{r}_{OA} \times (m \vec{v}_A)) \} \\ &= \vec{v}_A \times (m \vec{v}_A) + \vec{r}_{OA} \times \frac{d}{dt} \{ m \vec{v}_A \} \\ \boxed{\vec{H}_o = \vec{r}_{OA} \times \vec{F} = \vec{M}_o} \end{aligned}$$

ex:



$$+ \vec{T} \hat{e}_r = \frac{\vec{L}}{l}$$

$$mg \downarrow = F_g$$

$$\vec{\omega}_A = \underline{l \dot{\theta} \hat{e}_\theta}$$

$$\vec{H}_o = \vec{r}_{0A} \times (m \vec{\omega}_A) = (l \hat{e}_r) \times (m l \dot{\theta} \hat{e}_\theta) = m l^2 \dot{\theta} \hat{k}$$

Angular momentum equation:

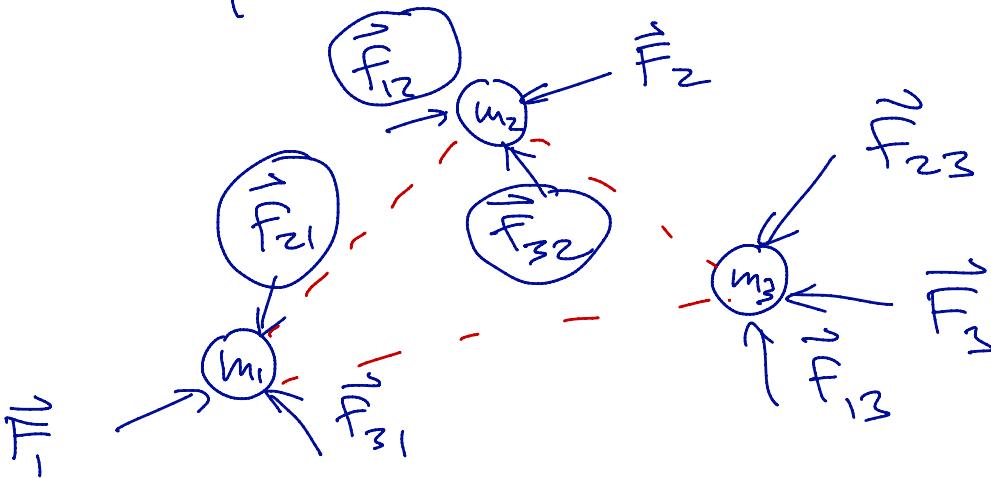
$$\dot{\vec{H}}_o = \frac{d}{dt} \{ m l^2 \dot{\theta} \hat{k} \} = m l^2 \ddot{\theta} \hat{k} = \vec{M}_o = \vec{r}_{0A} \times \vec{F}_g$$

$$\Rightarrow \ddot{\theta} = - \left(\frac{g}{l} \right) \sin \theta \quad \boxed{\text{colinear}}$$

~~+ $\vec{r}_{0A} \times \frac{\vec{L}}{l}$~~

Euler's Laws: Systems of Particles & Rigid Bodies

Consider a 3-particle system with forces acting between the particles and also external forces:



Equations of motion:

$$m_1 \vec{a}_1 = \vec{F}_1 + \vec{f}_{21} + \vec{f}_{31}$$

$$m_2 \vec{a}_2 = \vec{F}_2 + \vec{f}_{12} + \vec{f}_{32}$$

$$m_3 \vec{a}_3 = \vec{F}_3 + \vec{f}_{13} + \vec{f}_{23}$$

add the equations:

$$\underline{m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3}$$

Does the LHS represent some type of aggregate behavior?

Set $m = m_1 + m_2 + m_3 =$ total mass of system.

$$\begin{aligned} m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 &= m \left(\frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3}{m} \right) \\ &= m \frac{d^2}{dt^2} \left\{ \underbrace{\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m}}_{=: \vec{r}_c} \right\} \\ &= m \ddot{\vec{r}}_c \end{aligned}$$

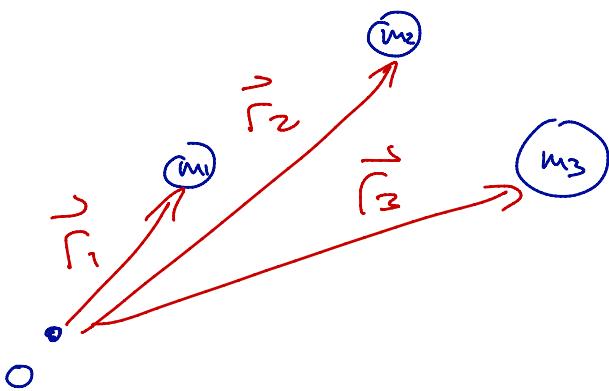
\Rightarrow Euler's first law

$\vec{F}_{\text{tot}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m \ddot{\vec{r}}_c$

Newton's 3rd
 $\vec{f}_{12} = -\vec{f}_{21}$
 $\vec{f}_{23} = -\vec{f}_{32}$
 $\vec{f}_{31} = -\vec{f}_{13}$

total
external
forces only.

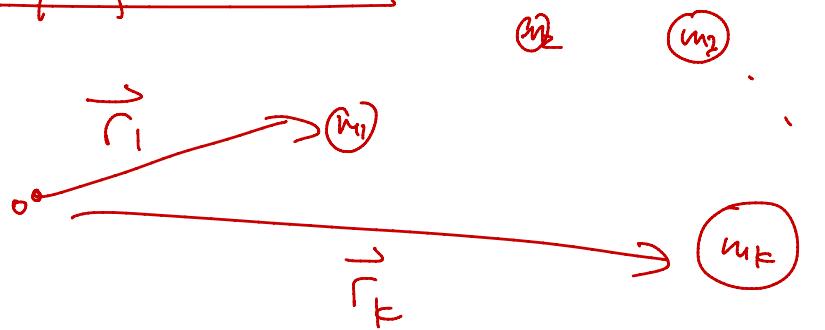
center of
mass of
the 3-part
system



$$\begin{aligned}\vec{r}_c &= \frac{1}{m} (m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3) \\ &= \left(\frac{m_1}{m}\right) \vec{r}_1 + \left(\frac{m_2}{m}\right) \vec{r}_2 + \left(\frac{m_3}{m}\right) \vec{r}_3\end{aligned}$$

vector weighted average

Many particles :



$$m \ddot{\vec{r}}_c = \vec{F}_{tot}$$

$$\vec{r}_c = \frac{1}{m} (m_1 \vec{r}_1 + \dots + m_k \vec{r}_k)$$

example: four masses

$$m_1 (2, 3)$$

$$(3.2, 1.6)$$

$$m_2 (4, 3)$$

$$m_4 (4, 1)$$



$$\begin{aligned}\vec{r}_1 &= 2\hat{i} + 3\hat{j}, & \vec{r}_2 &= 4\hat{i} + 3\hat{j} \\ \vec{r}_3 &= \hat{i} + \hat{j}, & \vec{r}_4 &= 4\hat{i} + \hat{j}\end{aligned}$$

$$\begin{aligned}m_1 &= 1 \text{ kg}; m_2 = 2 \text{ kg} \\ m_3 &= 2 \text{ kg}; m_4 = 5 \text{ kg} \\ \hline m &= 10 \text{ kg.}\end{aligned}$$

$$\begin{aligned}\vec{r}_c &= \frac{1}{10} \left((1)(2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 3\hat{j}) + (2)(\hat{i} + \hat{j}) \right. \\ &\quad \left. + (5)(4\hat{i} + \hat{j}) \right) \\ &= 3.2\hat{i} + 1.6\hat{j}\end{aligned}$$

Rigid Bodies: Continuum Bodies

