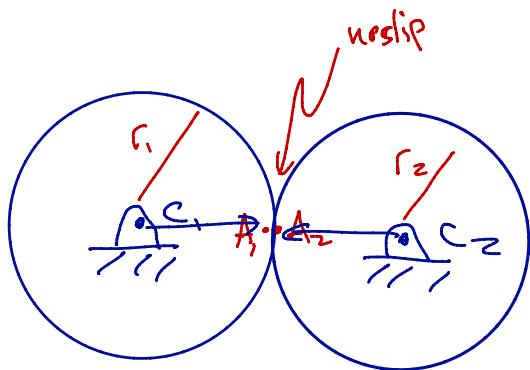


11/06/17

Lecture #26

Gears



$$\vec{v}_{A_1} = \vec{v}_{A_2}$$

Using the rigid body formula:

$$\vec{\omega}_1 \times \vec{r}_{C_1 A_1} = \vec{\omega}_2 \times \vec{r}_{C_2 A_2}$$

$$\Rightarrow \vec{r}_{C_1 A_1} = r_1 \hat{r}; \quad \vec{r}_{C_2} = -r_2 \hat{r}$$

$$\Rightarrow (\omega_1 \hat{k}) \times (r_1 \hat{r}) = (\omega_2 \hat{k}) \times (-r_2 \hat{r})$$

$$r_1 \omega_1 (\hat{k} \times \hat{r}) = -r_2 \omega_2 (\hat{k} \times \hat{r})$$

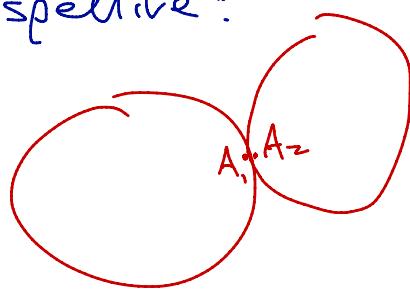
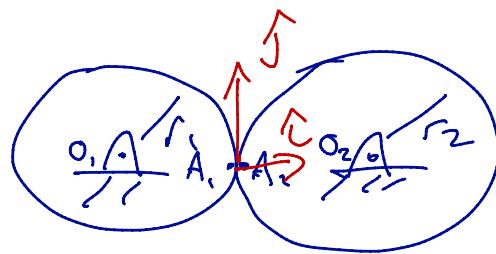
$$\text{or} \quad r_1 \omega_1 (\hat{k} \times \hat{r}) = -r_2 \omega_2 (\hat{k} \times \hat{r})$$

$$\text{differentiate: } r_1 \alpha_1(t) = -r_2 \alpha_2(t)$$

define \hat{k}
to be the unit
vector from
 C_1 to C_2 .

Standard gear equations: assume the centers of the disks don't move.

Let's look at this from another perspective:



$$\vec{\alpha}_{A_1} = \vec{\alpha}_1 \times \vec{r}_{o,A_1} - \omega_1^2 \vec{r}_{o,A_1}$$

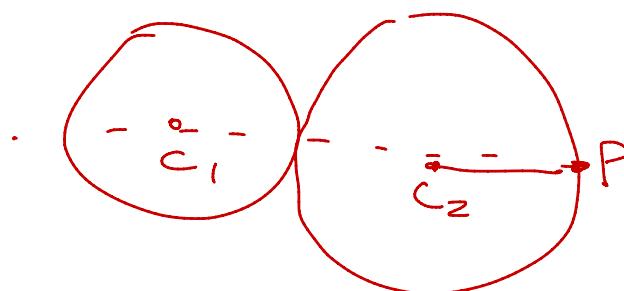
~~$\vec{\alpha}_{A_1} = r_1 \alpha_1 \hat{j} - \omega_1^2 r_1 \hat{k}$~~

~~$\vec{\alpha}_{A_2} = -r_2 \alpha_2 \hat{j} - \omega_2^2 r_2 \hat{k}$~~

$$r_1 \omega_1 = -r_2 \omega_2$$

$$r_1 \alpha_1 = -r_2 \alpha_2$$

Example:



$$r_1 = 2\text{m}$$

$$r_2 = 4\text{m}$$

$$\omega_1 = 2 \text{ rad/s}$$

$$\alpha_1 = -4 \text{ rad/s}^2$$

Find $\vec{\alpha}_P$:

$$r_1 \omega_1 = -r_2 \omega_2$$

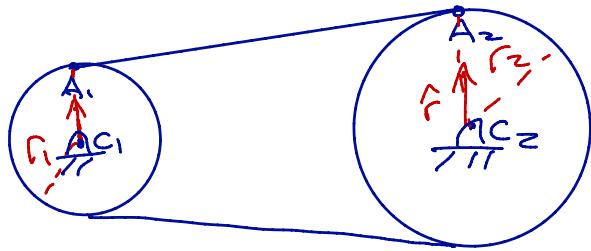
$$\underline{\omega_2 = \left(-\frac{r_1}{r_2}\right) \omega_1 = -1 \text{ rad/s}}$$

$$\underline{\alpha_2 = \left(-\frac{r_1}{r_2}\right) \alpha_1 = 2 \text{ rad/s}^2}$$

$$\vec{\alpha}_P = \vec{\alpha}_2 \times \vec{r}_{c_2 P} - \omega_2^2 \vec{r}_{c_2 P}$$

$$= (-2\hat{k}) \times (4\hat{i}) - (-1)^2 (4\hat{i}) = \underline{8\hat{j} - 4\hat{i}}$$

Chain Gears



$$\vec{\omega}_{A_1} = \vec{\omega}_{A_2}$$

Rigid body:

$$\vec{\omega}_1 \times \vec{r}_{C_1 A_1} = \vec{\omega}_2 \times \vec{r}_{C_2 A_2}$$

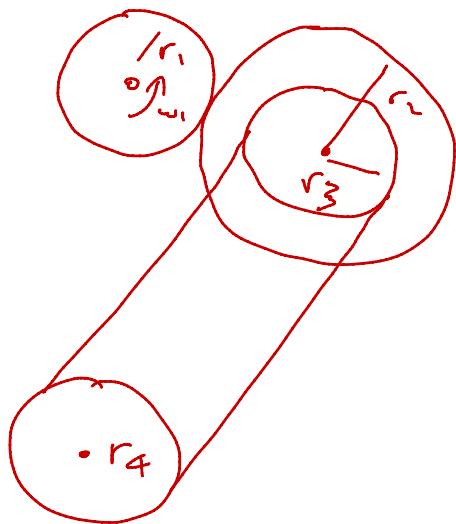
$$(\omega_1 \hat{k}) \times (r_1 \hat{r}) = (\omega_2 \hat{k}) \times (r_2 \hat{r})$$

$$r_1 \omega_1 (\hat{k} \times \hat{r}) = r_2 \omega_2 (\hat{k} \times \hat{r})$$

$$\frac{r_1 \omega_1(t)}{r_1 \alpha_1(t)} = \frac{r_2 \omega_2(t)}{r_2 \alpha_2(t)}$$

no negative;
not counter rotatly.

example:



Find: ω_4 in terms of ω_1

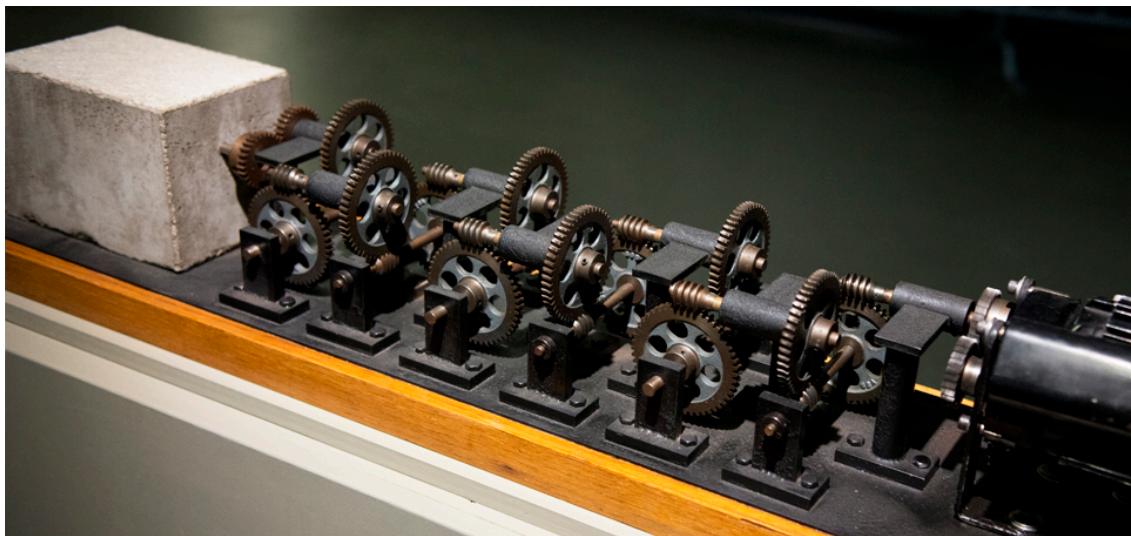
$$\omega_2 = -\left(\frac{r_1}{r_2}\right) \omega_1$$

$$\omega_4 = \left(\frac{r_2}{r_4}\right) \omega_2$$

$$\omega_4 = -\left(\frac{r_1}{r} \frac{r_3}{r_4}\right) \omega_1$$

Machine with Concrete

Arthur Ganson



12 gear-stages each stepping down the angular velocity by 50:1. One end is driven by an electric motor going 200 rpm. The other end is stuck in concrete.

Question: If you turn it on, what happens?

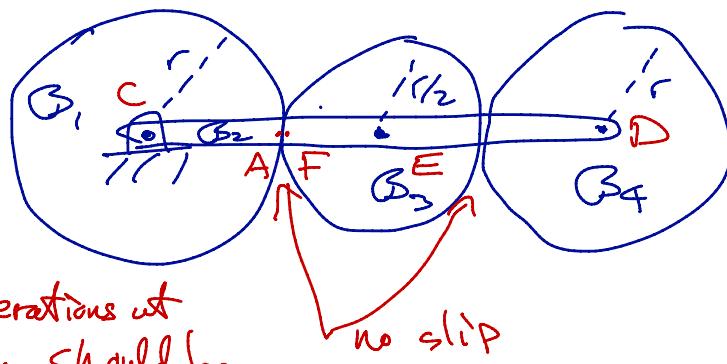
- (a) motor breaks (b) concrete breaks (c) gears break (d) nothing.



Gear ratio from motor to concrete

$$\left(\frac{1}{50}\right)^{12} \approx 10^{-21}$$

example:



Important:

tangential accelerations at points of rolling should be equal

Given: $\omega_1 = \omega_0$; $\omega_2 = -\omega_0$
 $\alpha_1 = \alpha_0$; $\alpha_2 = \alpha_0$

Find: ω_3 from a week ago $\omega_3 = -5\omega_0$
 α_3
 α_4

Solution: $\uparrow \cdot \vec{\alpha}_A = \uparrow \cdot \vec{\alpha}_F$

$$\vec{\alpha}_A = \vec{\alpha}_1 \times \vec{r}_{CA} - \omega_1^2 \vec{r}_{CA} = (\alpha_0 \hat{k}) \times (r \hat{i}) - \omega_0^2 (r \hat{i})$$

$$\vec{\alpha}_F = \vec{\alpha}_E + \vec{\alpha}_3 \times \vec{r}_{EF} - \omega_3^2 \vec{r}_{EF}$$

$$\vec{\alpha}_E = \vec{\alpha}_2 \times \vec{r}_{CE} - \omega_2^2 \vec{r}_{CE}$$

$$\Rightarrow \vec{\alpha}_F = \vec{\alpha}_2 \times \vec{r}_{CE} - \omega_2^2 \vec{r}_{CE} + \vec{\alpha}_3 \times \vec{r}_{EF} - \omega_3^2 \vec{r}_{EF}$$
$$= (\alpha_0) \times \left(\frac{3}{2}r\hat{i}\right) - \omega_0^2 \left(\frac{3}{2}r\hat{i}\right) + (\alpha_3 \hat{k}) \times \left(-\frac{r}{2}\hat{i}\right) - (-5\omega_0)^2 \left(\frac{r}{2}\hat{i}\right)$$

Now let's compute $\uparrow \cdot \vec{\alpha}_A = \uparrow \cdot \vec{\alpha}_F$:

$$\Rightarrow r\alpha_0 = \frac{3}{2}r\alpha_0 - \left(\frac{1}{2}\right)r\alpha_3$$

$$\boxed{\alpha_0 = \alpha_3} \quad \text{Ex similarly for } \underline{\alpha_4}.$$