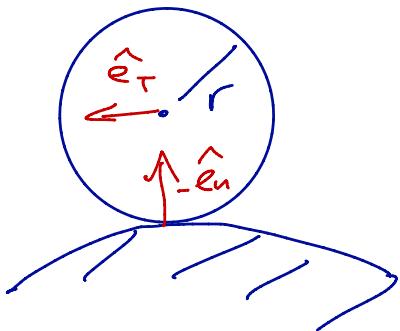
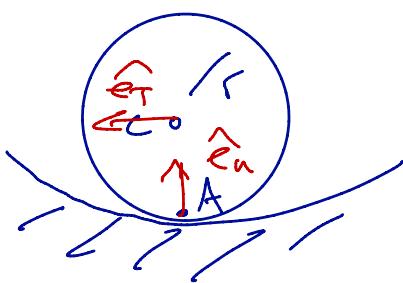


11/01/17

Lecture # 25

# Final Rolling Material

$\rho$  curvature of surface



$$\vec{v}_c = r\omega \hat{e}_T$$

$$\vec{a}_c = r\alpha \hat{e}_T + \frac{(rw)^2}{\rho-r} \hat{e}_n$$

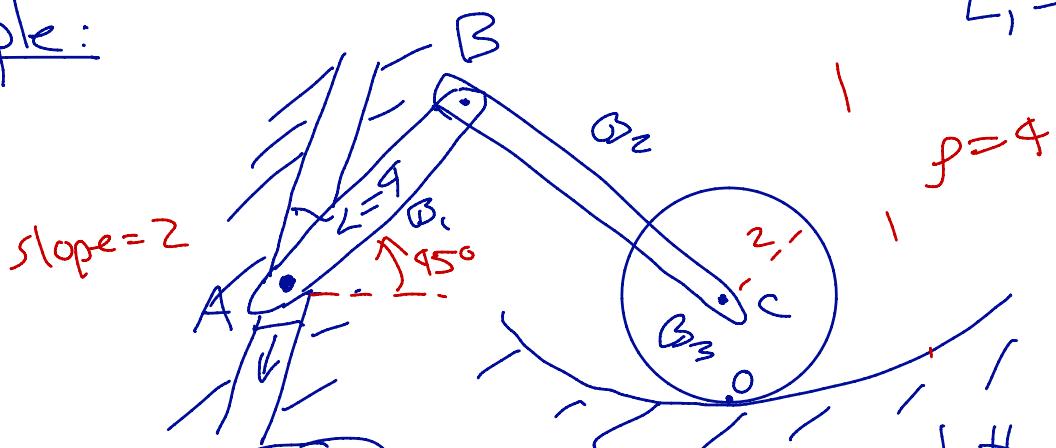
$$\vec{a}_A = \left(\frac{\rho}{\rho-r}\right) rw^2 \hat{e}_n$$

$$\vec{v}_c = r\omega \hat{e}_T$$

$$\vec{a}_c = r\alpha \hat{e}_T + \frac{(rw)^2}{\rho+r} \hat{e}_n$$

$$\vec{a}_A = -\left(\frac{\rho}{\rho+r}\right) rw^2 \hat{e}_n$$

example:



$$L_1 = L_2 = 4$$

$$\rho = 4$$

Given:

$$\omega_1 = \sqrt{2}$$

$$\alpha_1 = -1$$

$$\alpha_2 = 1$$

$$\|\vec{v}_c\| = 3$$

Find:

$$\vec{v}_B, \vec{a}_c$$

- bottom of cavity
- no slip
- remain in contact

Solution: velocity  $\vec{v}_B$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{AB}$$

$$= \vec{v}_A \frac{1}{\sqrt{5}} (\hat{i} + 2\hat{j}) + (\sqrt{2} \hat{k}) \times (2\sqrt{2} (\hat{i} + \hat{j}))$$

$$= \frac{\vec{v}_A}{\sqrt{5}} (\hat{i} + 2\hat{j}) + 4\hat{j} - 4\hat{i}$$

$$= \vec{v}_c + \vec{\omega}_2 \times \vec{r}_{cB} = -3\hat{i} + (\omega_2 \hat{k}) \times ((2\sqrt{2})(-\hat{i} + \hat{j}))$$

$$= -3\hat{i} + \omega_2 (-2\sqrt{2})(\hat{i} + \hat{j})$$

$$\Rightarrow \begin{aligned} \text{I: } \frac{\omega_A}{\sqrt{5}} - 4 &= -3 - 2\sqrt{2}\omega_2 \\ (\frac{1}{\sqrt{5}})\omega_A + 2\sqrt{2}\omega_2 &= 1 \end{aligned}$$

$$\begin{aligned} \omega &= \frac{1}{\sqrt{5}}\omega_A \\ \omega &= 2\sqrt{2}\omega_2 \end{aligned}$$

$$\begin{aligned} \text{II: } \frac{2}{\sqrt{5}}\omega_A + 4 &= -2\sqrt{2}\omega_2 \quad \Rightarrow \omega_A = -5\sqrt{5} \\ \frac{2}{\sqrt{5}}\omega_A + 2\sqrt{2}\omega_2 &= -4 \quad \omega_2 = 3/\sqrt{2} \end{aligned}$$

Sub back in for  $\vec{v}_B$  using either  $\omega_A$  or  $\omega_2$

$$\Rightarrow \vec{v}_B = -9\hat{i} - 6\hat{j}.$$

Acceleration  $\vec{a}_c$ :

$$\begin{aligned} \vec{a}_B &= \vec{a}_A + \frac{1}{\sqrt{5}}(2\hat{i} + 2\hat{j}) + \vec{\alpha}_2 \times \vec{r}_{AB} \\ &= \vec{a}_c + \vec{\alpha}_2 \times \vec{r}_{CB} - \omega_2^2 \vec{r}_{CR} \end{aligned}$$

$$\vec{a}_c = r\dot{\alpha}_3 \hat{e}_T + \frac{(r\omega_3)^2}{r} \hat{e}_n$$

We need  $\omega_3$ :  $\vec{v}_c = -3\hat{i} = \vec{\omega}_3 \times \vec{r}_{oc}$   
 $= (\omega_3 \hat{k}) \times (2\hat{j}) = -2\omega_3 \hat{i}$

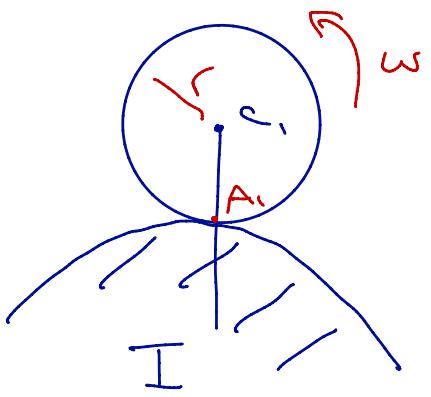
$$\Rightarrow \omega_3 = 3/2 \text{ rad/sec}$$

$$\vec{a}_c = 2\alpha_3(-\hat{i}) + \left(\frac{2(3/2)^2}{9-2}\right) \hat{j}$$

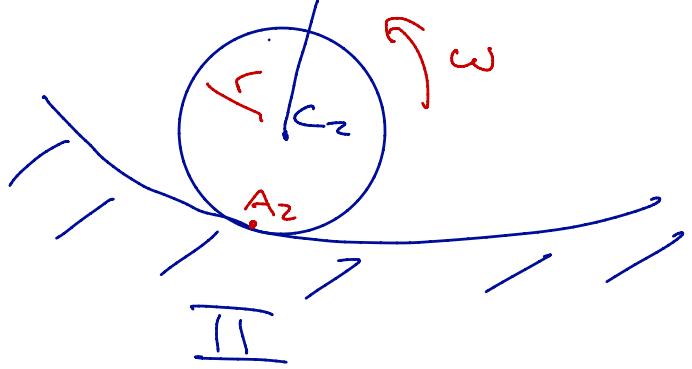
Have two linear equations in  $\alpha_3$  and  $\alpha_A$ .

OK

①



I



II

In which of the pictures is the radius of curvature followed by the center point C. largest?

(a)

(b)

(c) can't say.

(2) Which of the following true.

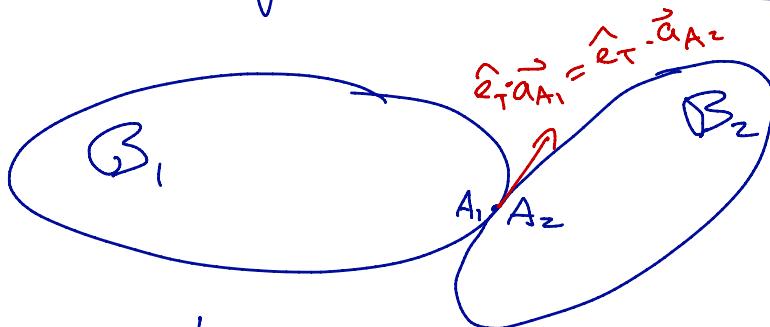
(a)  $\|\vec{a}_{A_1}\| > \|\vec{a}_{A_2}\|$

(b)  $\|\vec{a}_{A_1}\| < \|\vec{a}_{A_2}\|$

(c)  $\|\vec{a}_{A_1}\| = \|\vec{a}_{A_2}\|$

(d) can't say

### No-Slip Rolling with two free bodies



no slip means

$$\vec{v}_{A_1} = \vec{v}_{A_2}$$

What about acceleration?

$$\vec{a}_{A_1} = \vec{a}_{A_2} \quad \text{☒}$$

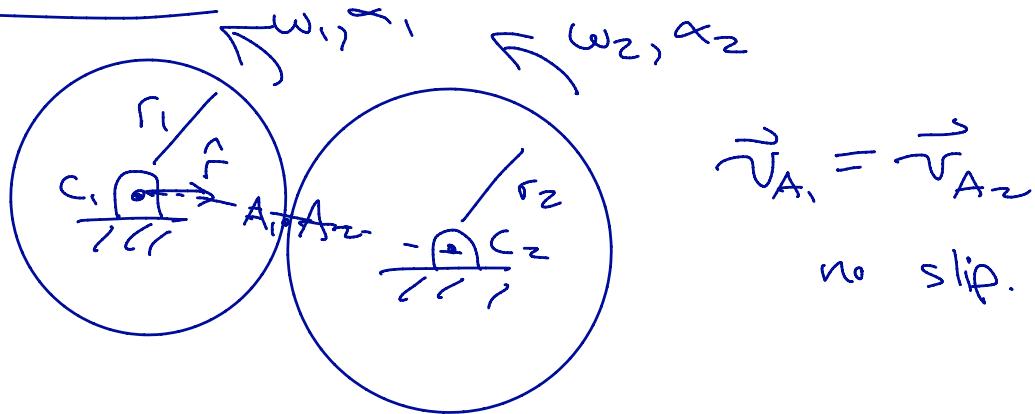
$\vec{a}_A$  = in the  $\hat{e}_n$  direction  
nonzero if  $\omega \neq 0$

$$\vec{a}_o = \vec{0}$$

Conclude

$$1) \quad \hat{e}_T \cdot \vec{a}_{A_1} = \hat{e}_T \cdot \vec{a}_{A_2}$$

## Gear Trains



Rigid body formula:

$$\vec{\omega}_1 \times \vec{r}_{C_1 A_1} = \vec{\omega}_2 \times \vec{r}_{C_2 A_2}$$

$$(\omega_1 \hat{k}) \times (r_1 \hat{r}) = (\omega_2 \hat{k}) \times (-r_2 \hat{r})$$

$$\omega_1 r_1 (\hat{k} \times \hat{r}) = -\omega_2 r_2 (\hat{k} \times \hat{r})$$

→  $r_1 \dot{\omega}_1 (+) = -r_2 \dot{\omega}_2 (+)$