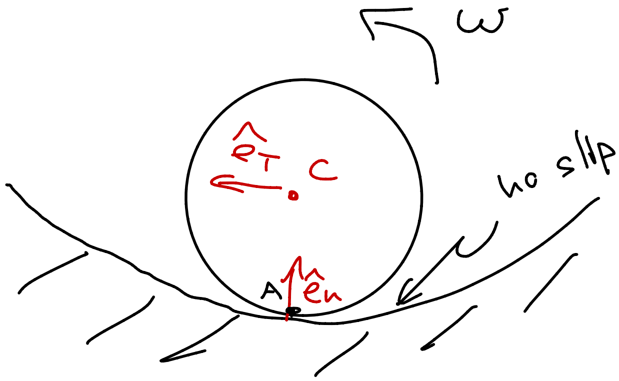
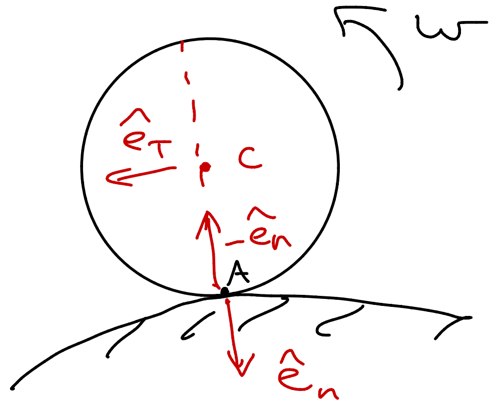
10-30-17

Lecture #24

Curved Surfaces and Rolling (cont'd)



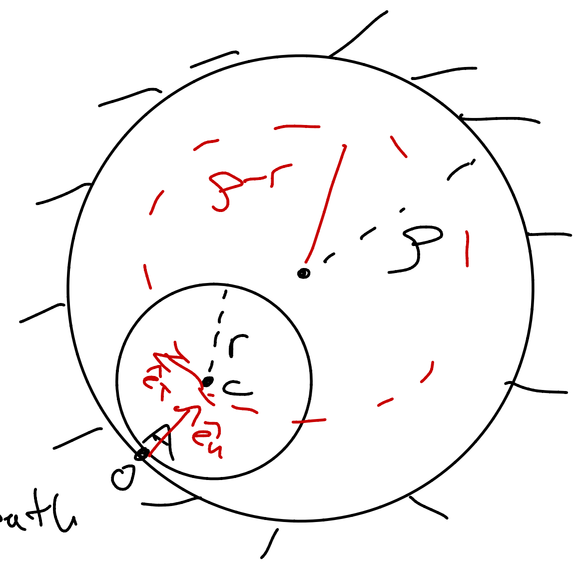


$\vec{v}_A = \vec{0}$

In both these scenarios we have

$$\vec{v}_c = \vec{\omega} \times \vec{r}_{Ac} = r\omega\,\hat{e}_T$$

Acceleration:



inside of curve.

Know that point C follows a circular path of radius $\rho - r$.

$$\Longrightarrow \vec{a}_c = r\alpha\,\hat{e}_T + \frac{(r\omega)^2}{\rho - r}\,\hat{e}_n$$

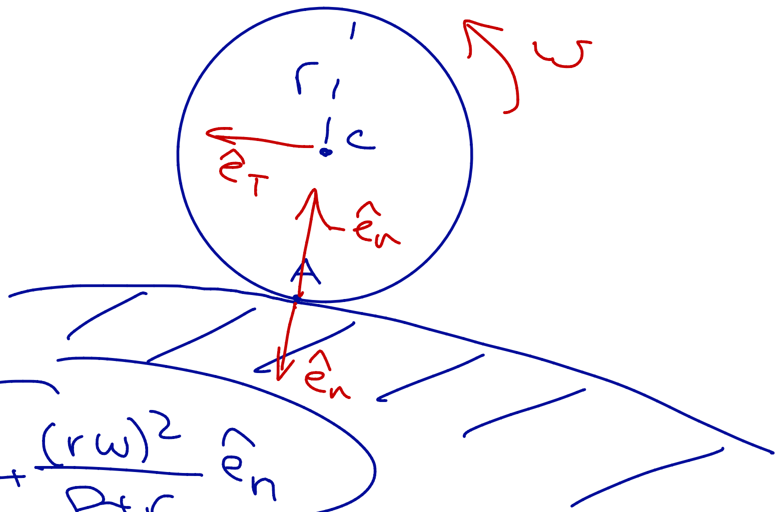
Generally:

$$\vec{a}_c = r\alpha\,\hat{e}_T + \frac{(r\omega)^2}{\rho - r}\,\hat{e}_n$$

ρ = radius of curve at A.

This gave us $\vec{a}_A = \left(\frac{\rho r \omega^2}{\rho - r}\right)\hat{e}_n$

Outside of curve



$$\vec{a}_C = r\alpha\,\hat{e}_T + \frac{(r\omega)^2}{\rho + r}\,\hat{e}_n$$

Let's derive

$$\vec{a}_A = \vec{a}_C + \vec{\alpha}\times\vec{r}_{CA} - \omega^2\vec{r}_{CA}$$

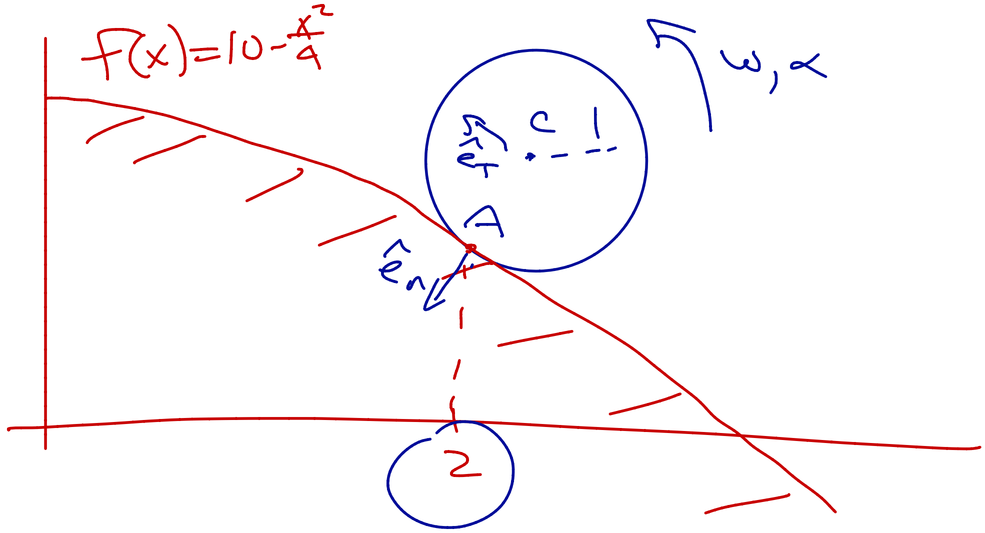
$$= \cancel{r\alpha\,\hat{e}_T} + \frac{(r\omega)^2}{\rho+r}\,\hat{e}_n + \overbrace{\cancel{(\alpha\hat{k})\times(r\hat{e}_n)}}^{\hat{k}\times\hat{e}_n = -\hat{e}_T} - \omega^2(r\hat{e}_n)$$

$$= \frac{(r\omega)^2}{\rho+r}\,\hat{e}_n - r\omega^2\,\hat{e}_n$$

$$= \left(\frac{-\rho r\omega^2}{\rho+r}\right)\hat{e}_n = \underline{-\left(\frac{\rho}{\rho+r}\right)(r\omega^2)\,\hat{e}_n}$$

If the surface is flat $\vec{a}_A = -r\omega^2\hat{e}_n$, so the outside of a curve tends to lessen accelerations.

example:



Given: $\omega = 10$, $\alpha = -1$

Find: velocities and accelerations of points A & C.

Solution: velocities: $\vec{v}_A = 0$

$$f'(x) = -\frac{1}{2}x \Longrightarrow f'(2) = -1$$

$$\hat{e}_T = \frac{1}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath}); \quad \hat{e}_n = \left(\frac{-1}{\sqrt{2}}\right)(\hat{\imath}+\hat{\jmath})$$

$$\vec{v}_c = \vec{\omega} \times \vec{r}_{Ac} = (10\hat{k}) \times \left(\frac{1}{\sqrt{2}}\right)(\hat{\imath}+\hat{\jmath})$$

$$= \frac{10}{\sqrt{2}}(\hat{\jmath}-\hat{\imath}) \qquad \boxed{\checkmark}$$

acceleration: $\alpha = -1$, $\omega = 10$, $r = 1$

have $$\vec{a}_A = -\left(\frac{\rho r \omega^2}{\rho + r}\right)\hat{e}_n$$

We need ρ: recall the formula $$\frac{1}{\rho} = \frac{|f''(x)|}{(1+f'(x)^2)^{3/2}}$$

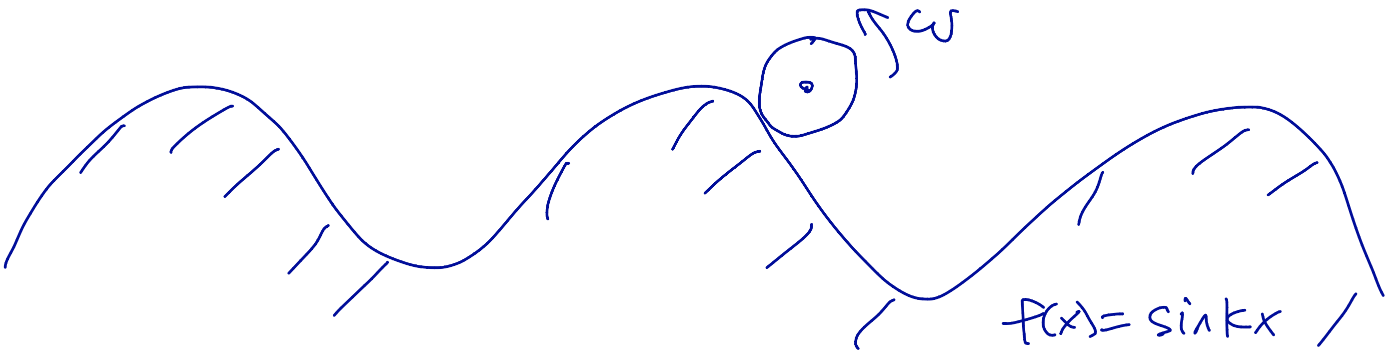
$$f''(x) = -\frac{1}{2}; \quad f'(2) = -1$$

$$\Longrightarrow \rho = 4\sqrt{2}$$

$$\vec{a}_A = \left(\frac{4\cancel{\sqrt{2}}(10)^2}{4\sqrt{2}+1}\right)\left(\frac{1}{\cancel{\sqrt{2}}}\right)(\hat{\imath}+\hat{\jmath})$$

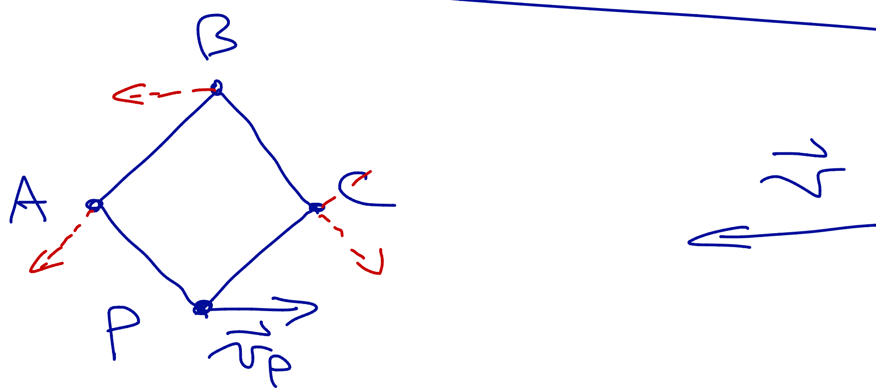
To get $\vec{a}_c = r\alpha\,\hat{e}_T + \frac{(r\omega)^2}{\rho + r}\,\hat{e}_n$

example:



iClickr:

(o) Given



Which of the other points could have velocity $\vec{v}$?

(a) A

(b) B

(c) C

(d) none is possible.

$$\vec{v}_Q = \vec{v}_P + \underbrace{\vec{\omega}\times\vec{r}_{PQ}}_{\text{perpendicular to } \vec{r}_{PQ}}$$

①

Given

Which of the following could represent the $\vec{a}_c$ and $\vec{a}_A$?

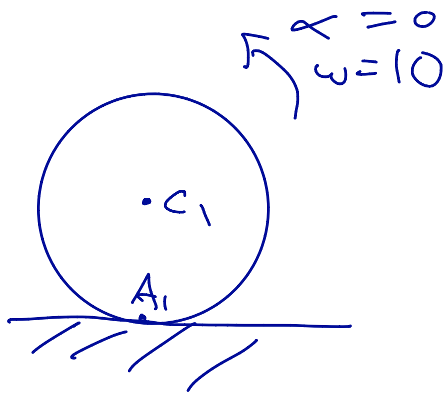
(a) $\vec{a}_c \nearrow$ $\quad\uparrow \vec{a}_A$

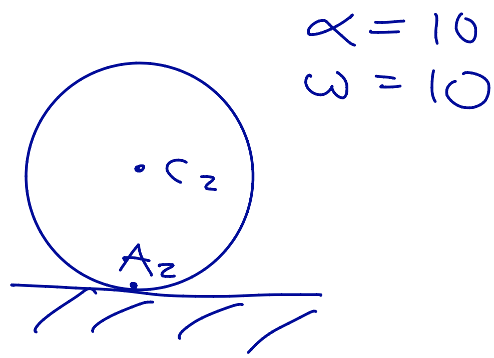
(b) $\vec{a}_c \rightarrow$ $\quad\downarrow \vec{a}_A$

(c) $\leftarrow \vec{a}_c$ $\quad\uparrow \vec{a}_A$ ☑

(d) $\uparrow \vec{a}_c$ $\quad\vec{a}_A \rightarrow$

②





(a) $\|\vec{a}_{A_1}\| > \|\vec{a}_{A_2}\|$

(b) $\|\vec{a}_{A_1}\| < \|\vec{a}_{A_2}\|$

(c) $\|\vec{a}_{c_1}\| > \|\vec{a}_{c_2}\|$

☑ (d) $\|\vec{a}_{c_1}\| < \|\vec{a}_{c_2}\|$

(e) $\vec{a}_{A_1} \neq \vec{a}_{A_2}$