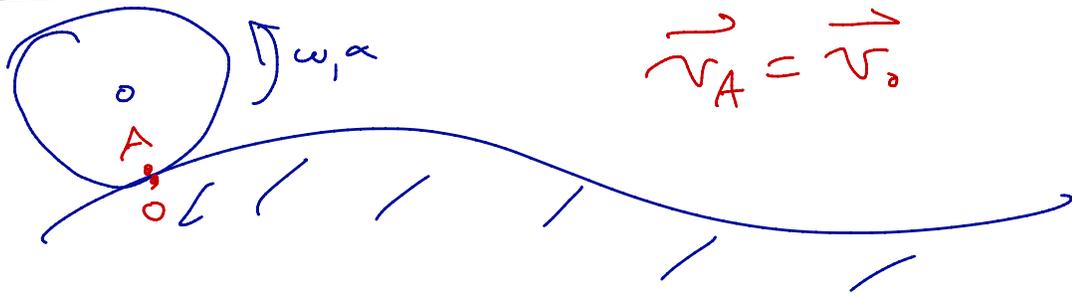


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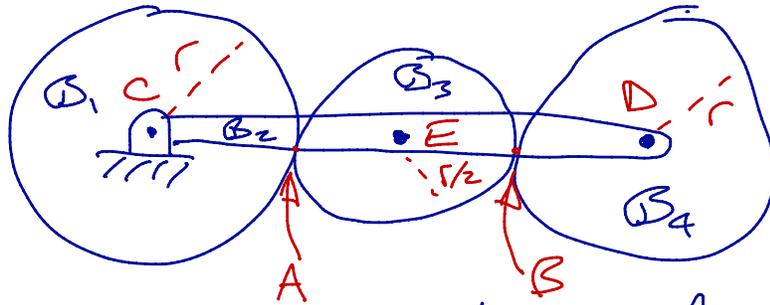
Lecture # 23

iClickr Monday

Rolling on a Curved Surface



example:



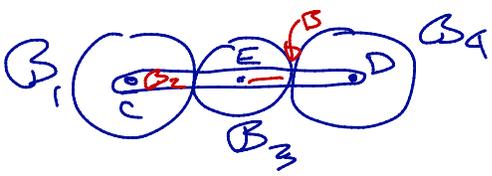
Given: the physical configuration and that $\vec{\omega}_1 = \omega_0 \hat{k}$ and $\vec{\omega}_2 = -\omega_0 \hat{k}$.

Find: the angular velocities of gears B_3 and B_4 .

Solution: from B_1 : $\vec{v}_A = \vec{v}_O + \vec{\omega}_1 \times \vec{r}_{OA}$
 $= (\omega_0 \hat{k}) \times (r \hat{c}) = r \omega_0 \hat{j}$ ✓

From B_3 : $\vec{v}_A = \vec{v}_E + \vec{\omega}_2 \times \vec{r}_{EA}$
 $= \vec{v}_E + \vec{\omega}_2 \times \vec{r}_{CE} + \vec{\omega}_3 \times \vec{r}_{EA}$
rigid body B_2
 $= (-\omega_0 \hat{k}) \times (\frac{3}{2} r \hat{c}) + (\omega_3 \hat{k}) \times (-\frac{r}{2} \hat{c})$
 $= -\frac{3}{2} r \omega_0 \hat{j} - \frac{1}{2} r \omega_3 \hat{j}$

Equating $r \omega_0 \hat{j} = -\frac{3}{2} r \omega_0 \hat{j} - \frac{1}{2} r \omega_3 \hat{j}$
 $\frac{5}{2} \omega_0 r \hat{j} = -\frac{1}{2} r \omega_3 \hat{j}$
 $\Rightarrow \boxed{\omega_3 = -5 \omega_0}$



Similarly for ω_4 :

$$\vec{v}_D = \vec{\omega}_2 \times \vec{r}_{CD} = (-\omega_0 \hat{k}) \times (3r \hat{i}) = \underline{-3r\omega_0 \hat{j}}$$

B₃ rigid body: $\vec{v}_B = \vec{v}_E + \vec{\omega}_3 \times \vec{r}_{EB}$ $\omega_3 = -5\omega_0$

$$= (-\omega_0 \hat{k}) \times \left(\frac{3}{2}r \hat{i}\right) + (-5\omega_0 \hat{k}) \times \left(\frac{r}{2} \hat{i}\right)$$

$$= -\frac{3}{2}r\omega_0 \hat{j} - \frac{5}{2}r\omega_0 \hat{j}$$

$$= \underline{-4r\omega_0 \hat{j}}$$

B₄ rigid body:

$$\vec{v}_B = \vec{v}_D + \vec{\omega}_4 \times \vec{r}_{DB}$$

$$= -3r\omega_0 \hat{j} + (\omega_4 \hat{k}) \times (-r \hat{i})$$

$$= -3r\omega_0 \hat{j} - r\omega_4 \hat{j}$$

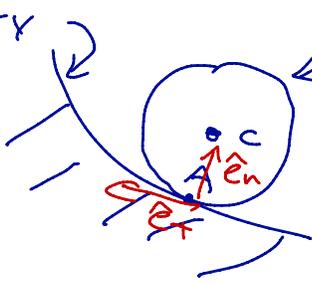
$$\underline{-4r\omega_0 \hat{j}} = -3r\omega_0 \hat{j} - r\omega_4 \hat{j}$$

$$r\omega_4 \hat{j} = r\omega_0 \hat{j}$$

$$\Rightarrow \underline{\omega_4 = \omega_0}$$

Review: Velocity

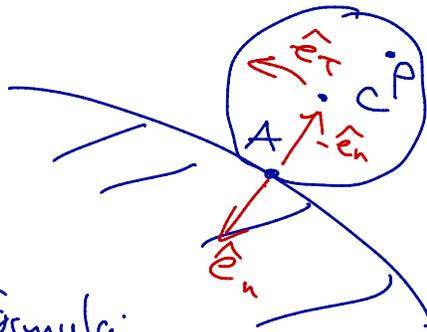
arbitrary curve



circular

$$\vec{v}_A = \vec{0}$$

no slip



$$\vec{r}_{Ac} = r \hat{e}_n$$

Use rigid body formula:

$$\vec{v}_c = \vec{\omega} \times \vec{r}_{Ac}$$

$$\vec{r}_{Ac} = -r \hat{e}_n$$

Furthermore,
$$\vec{v}_p = \vec{\omega} \times \vec{r}_{Ap}$$

Acceleration: To get better intuition let's consider rolling on the inside of a circular curve.

$$\vec{v}_c = r\omega \hat{e}_t$$

$$\vec{a}_c = \frac{d}{dt} \{ r\omega \hat{e}_t \}$$

$$= r\dot{\omega} \hat{e}_t + r\omega \frac{d}{dt} \{ \hat{e}_t \}$$

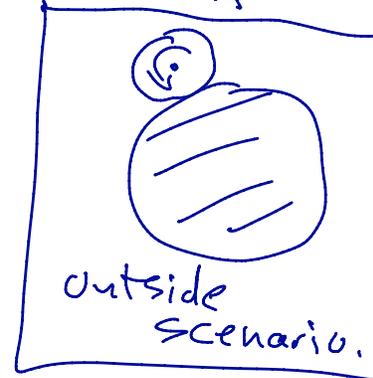
$$= r\alpha \hat{e}_t + r\omega \|\dot{\hat{e}_t}\| \frac{1}{R} \hat{e}_n$$

$$= r\alpha \hat{e}_t + \frac{(r\omega)^2}{R} \hat{e}_n$$

radius of curvature of path of C.



$$\vec{v}_A = \vec{0}$$



outside scenario.

What is R??

$$R = \rho - r$$

$$\Rightarrow \vec{a}_c = r\alpha \hat{e}_t + \frac{(r\omega)^2}{\rho - r} \hat{e}_n$$

General situation:

$$\vec{a}_c = r\alpha \hat{e}_T + \frac{(r\omega)^2}{\rho - r} \hat{e}_n$$



Note: this is

$$\dot{s} \hat{e}_T + \frac{\dot{s}^2}{(\rho - r)} \hat{e}_n$$

ρ is the radius of curvature of the surface at point A.

We can now use the rigid body formula to compute the accelerations anywhere on the disc. Let's look at contact point A:

$$\begin{aligned} \vec{a}_A &= \vec{a}_c + \vec{\alpha} \times \vec{r}_{cA} - \omega^2 \vec{r}_{cA} \\ &= r\alpha \hat{e}_T + \frac{(r\omega)^2}{\rho - r} \hat{e}_n + (\alpha \hat{k}) \times (-r \hat{e}_n) - \omega^2 (-r \hat{e}_n) \\ &= \frac{(r\omega)^2}{\rho - r} \hat{e}_n + r\omega^2 \hat{e}_n = \left(\frac{\rho r \omega^2}{\rho - r} \right) \hat{e}_n \end{aligned}$$

no tangential component!