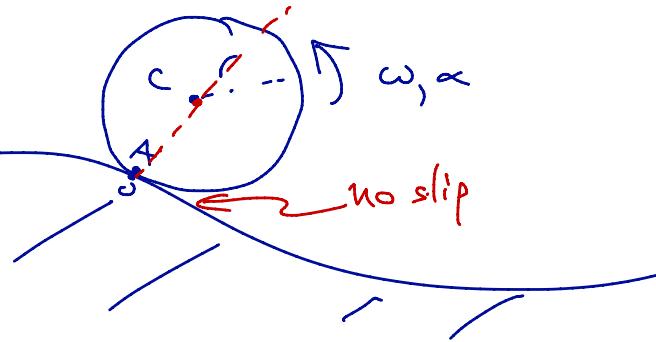


10-25-17

Lecture #22

No-Slip on Curved Surface



constraint:

$$\vec{v}_A = \vec{v}_o = \vec{0}$$

$$\vec{v}_c = \vec{\omega} \times \vec{r}_{Ac} \rightarrow \text{rigid body.}$$

Inside of curve:

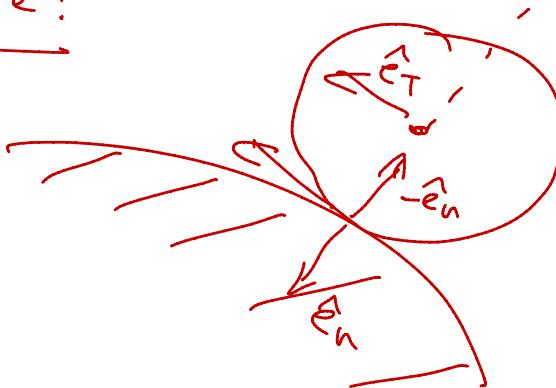
$$\Rightarrow \vec{r}_{Ac} = r \hat{e}_n$$

$$\vec{v}_c = \vec{\omega} \times \vec{r}_{Ac}$$

$$= \vec{\omega} \hat{k} \times r \hat{e}_n = rw \hat{k} \times \hat{e}_n = rw \hat{e}_t$$



Outside of curve:

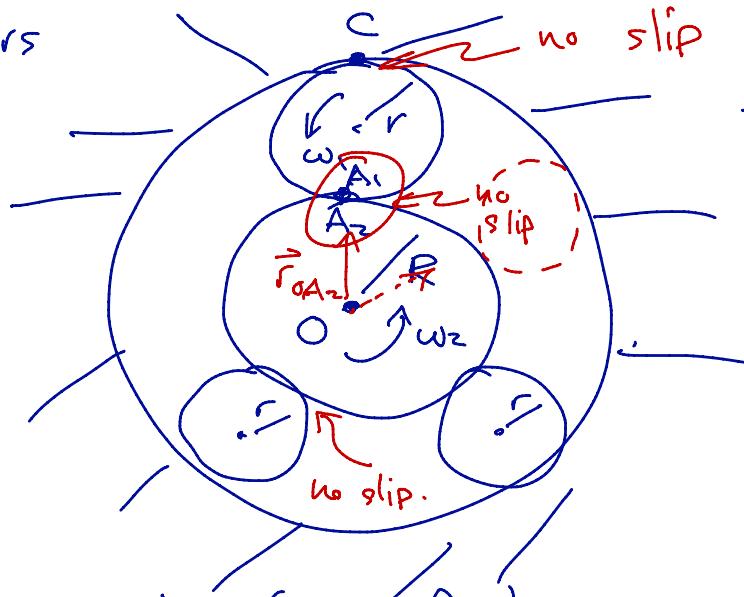


$$\vec{r}_{Ac} = -r \hat{e}_n$$

$$\vec{v}_c = (\vec{\omega} \hat{k}) \times (-r \hat{e}_n)$$

$$= rw \hat{e}_t$$

example: gears



Find: relationship between α_1 and α_2



$$\begin{aligned}\vec{\omega}_{A_1} &= \vec{\omega}_1 \times \vec{r}_{CA} = \vec{\omega}_1 \times (-2r \hat{r}_{OA}) \\ &= \vec{\omega}_1 \hat{k} \times (-2r \hat{r}_{OA}) \\ &= \vec{\omega}_{A_2} = \vec{\omega}_2 \times \vec{r}_{OA_2} = \vec{\omega}_2 \times R \hat{r}_{OA}\end{aligned}$$

$$\begin{aligned}\vec{\omega}_1 &= \omega_1 \hat{k} \\ \vec{\omega}_2 &= \omega_2 \hat{k}\end{aligned}$$

$$\Rightarrow -2r\omega_1(\hat{k} \times \hat{r}_{OA}) = R\omega_2(\hat{k} \times \hat{r}_{OA})$$

This holds for all time.

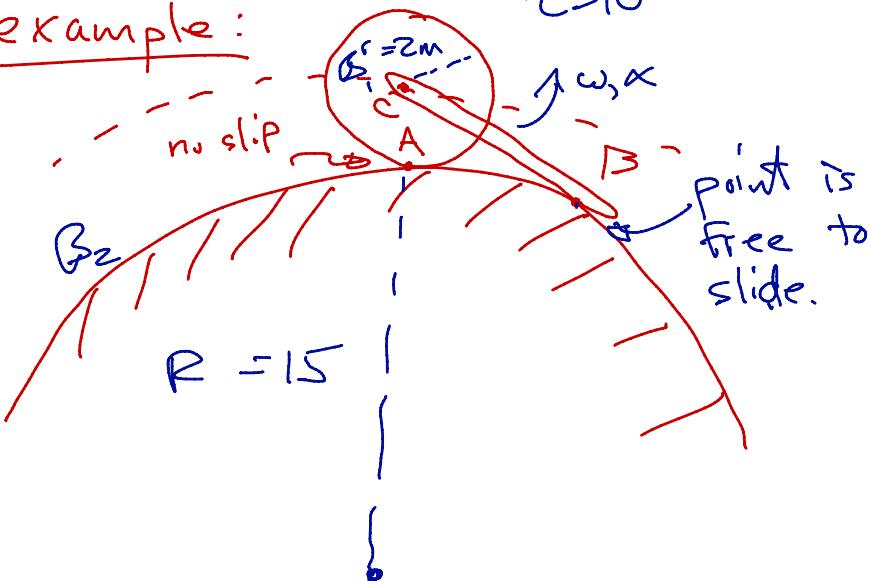
$$-2r\omega_1(t) = R\omega_2(t)$$

\hat{t} means at all times.

$$\Rightarrow -2r\dot{\omega}_1(t) = R\dot{\omega}_2(t)$$

$$\underline{\alpha_1 = \left(-\frac{R}{2r}\right)\alpha_2}$$

example:



Given: the configuration and speed of C is $0.1t^2$ m/s.

Find: the acceleration of point B

Gameplan: (I) compute \vec{v}_c and \vec{a}_c using new formulas for velocity and acceleration.

(II) relate the velocity and acceleration of point B to that of point C.

o further use the path information.

Step I: $\|\vec{v}_c\| = 0.1t^2 \Rightarrow \|\vec{v}_c\| = 10$ m/s.

$$\vec{v}_c = 10 \hat{e}_r$$

$$\vec{a}_c = \frac{d}{dt} \left\{ \|\vec{v}_c\| \right\} \hat{e}_r + \frac{\|\vec{v}_c\|^2}{R+r} \hat{e}_n$$

$$\therefore \ddot{s} = \frac{d}{dt} \left\{ 0.1t^2 \right\} = 0.2t \quad \Rightarrow \quad \vec{a}_c = 2 \hat{e}_r - \left(\frac{100}{17} \right) \hat{e}_n$$

$$\ddot{s}|_{t=10} = 2$$

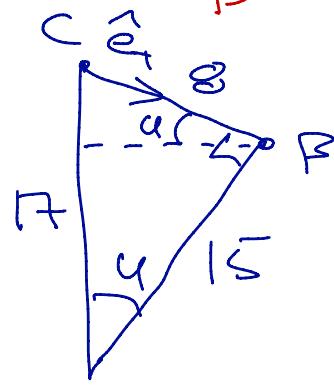
Step II:

Here: \hat{e}_T' simply means \hat{e}_T^B , not derivative.

$$\vec{v}_B = \vec{v}_C + \vec{\omega} \times \vec{r}_{CB}$$

$$\vec{v}_B = \vec{v}_B(\hat{e}_T) = v_B \left(\left(\frac{15}{17}\right)\hat{i} - \left(\frac{8}{17}\right)\hat{j} \right)$$

→ Solve to get v_B & ω
and hence \vec{v}_B .



Now for acceleration

$$\vec{a}_B = \vec{a}_c + \alpha \times \vec{r}_{CB} - \omega^2 \vec{r}_{CB}$$

set equal

$$\vec{a}_B = \frac{d}{dt} \left\{ \vec{v}_B \right\} \hat{e}_T' + \frac{11v_B^2 R}{R} \hat{e}_n'$$

$$\hat{e}_n = -\left(\frac{8}{17}\right)\hat{i} + \left(\frac{15}{17}\right)\hat{j}$$

Solve for α and \ddot{s}_B and sub back to get

$$\vec{a}_B = -0.889\hat{i} - 5.41\hat{j}$$