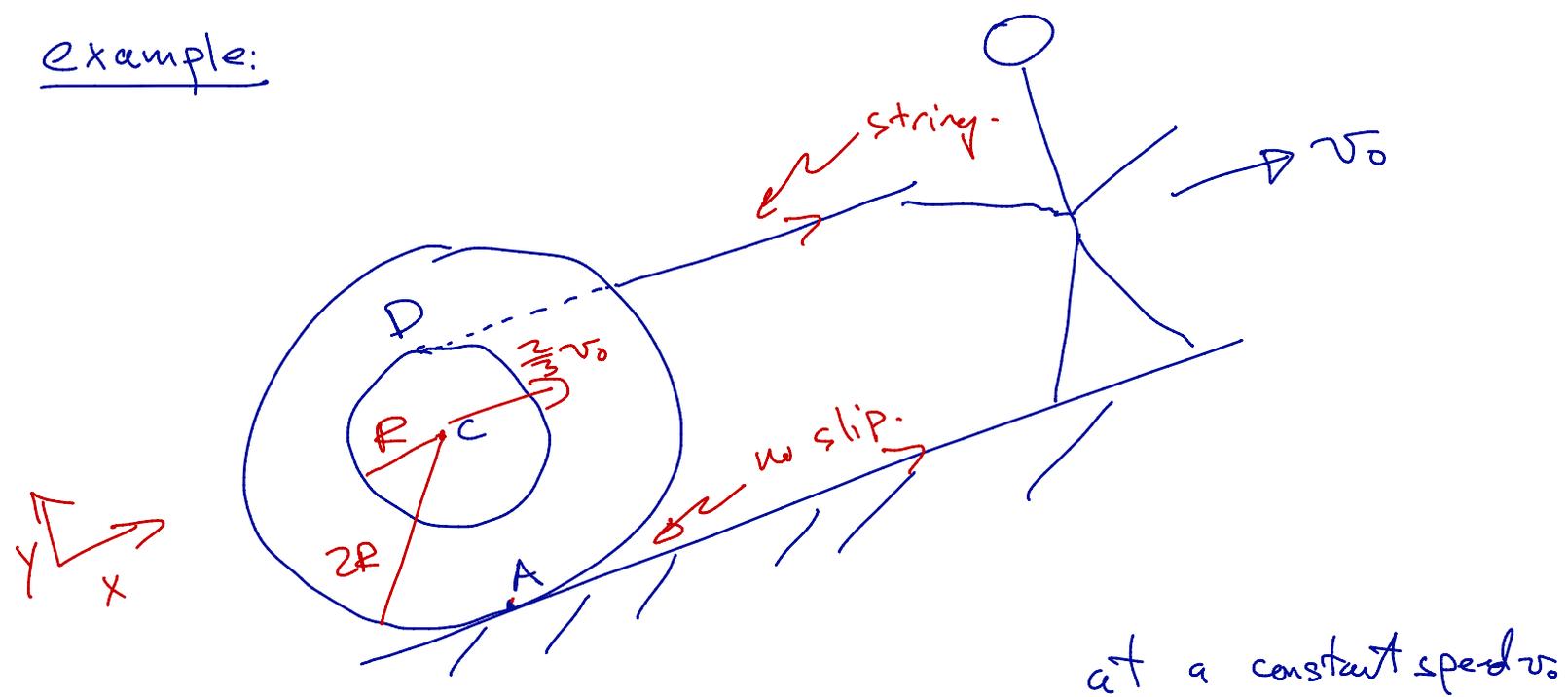


10-23-17

Lecture #21



example:



at a constant speed  $v_0$

A woman walks up an incline plane, pulling a spool. At what speed does the spool follow?

Solution:  $\vec{v}_A = \vec{0}$  and  $\vec{v}_D = v_0 \hat{c}$

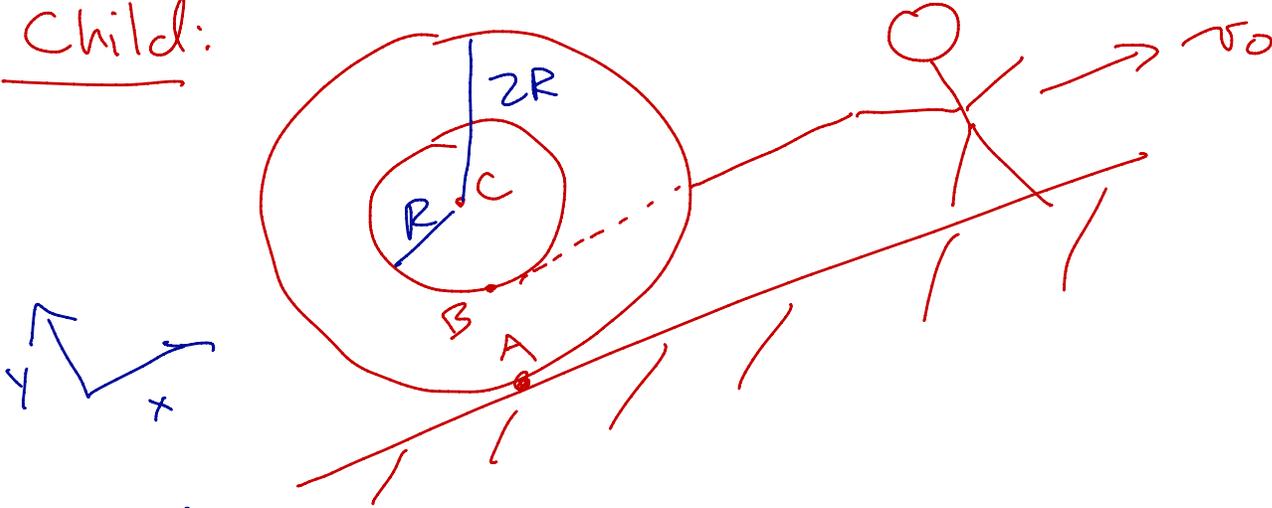
Let find  $\omega$ :  $\vec{v}_D = \vec{v}_A + \vec{\omega} \times \vec{r}_{AD}$  (rigid body)

$$v_0 \hat{c} = \vec{0} + (\omega \hat{k}) \times (3R \hat{j}) = -3\omega R \hat{c}$$

$$\Rightarrow \omega = -\frac{v_0}{3R}$$

Now  $\vec{v}_C = \vec{v}_A + \vec{\omega} \times \vec{r}_{AC} = \left(-\frac{v_0 \hat{k}}{3R}\right) \times (2R \hat{j}) = \frac{2}{3} v_0 \hat{c}$

Child:



Same basic analysis:

$$\vec{v}_A = \vec{0}, \quad \vec{v}_B = v_0 \hat{z}$$

$$\vec{v}_C = \vec{\omega} \times \vec{r}_{Ac} \quad \vec{v}_C = \vec{v}_B + \vec{\omega} \times \vec{r}_{Bc}$$

$$(\omega \hat{k}) \times (2R \hat{j}) = v_0 \hat{z} + (\omega \hat{k}) \times R \hat{j}$$

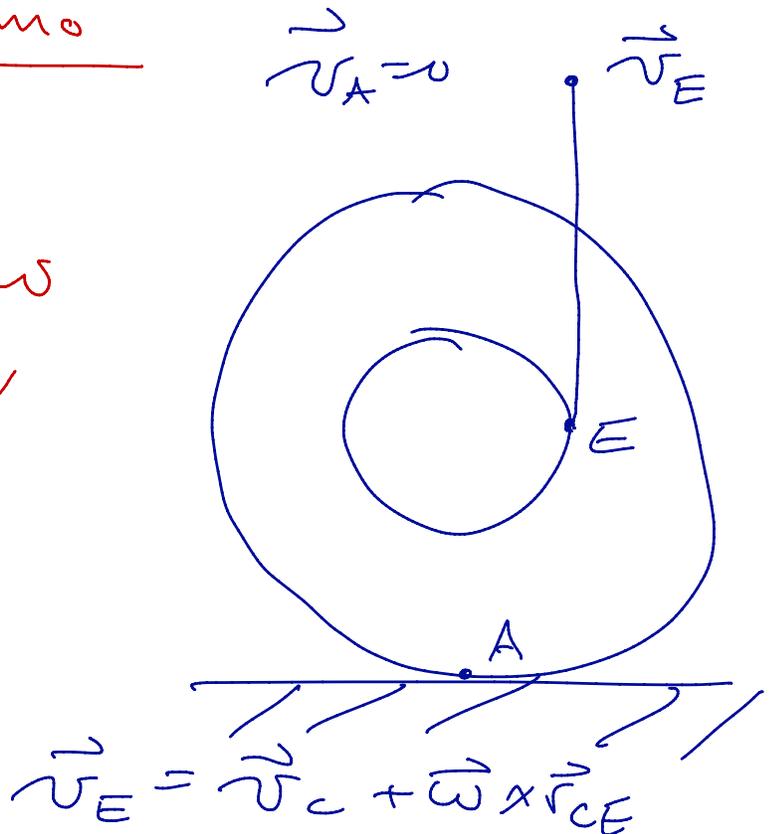
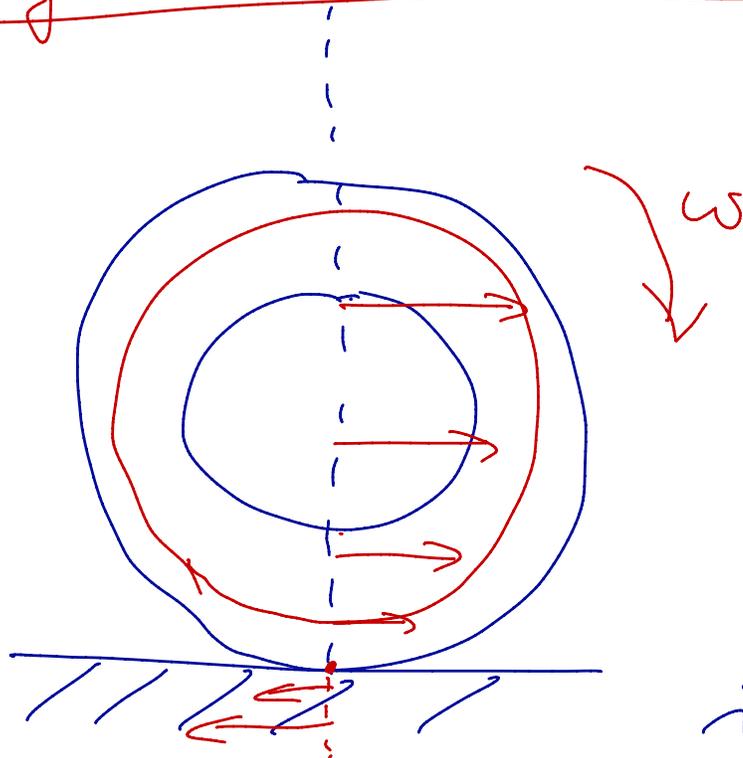
$$-2\omega R \hat{i} = v_0 \hat{z} - R\omega \hat{i}$$

$$\Rightarrow -\omega R \hat{i} = v_0 \hat{z} \Rightarrow \omega = -\frac{v_0}{R} \hat{i}$$

$$\Rightarrow \vec{v}_C = 2v_0 \hat{z} !!$$

Danger

Big Picture on Demo



$$\vec{v}_E = \vec{v}_C + \vec{\omega} \times \vec{r}_{CE}$$

example:

Consider the following setup.

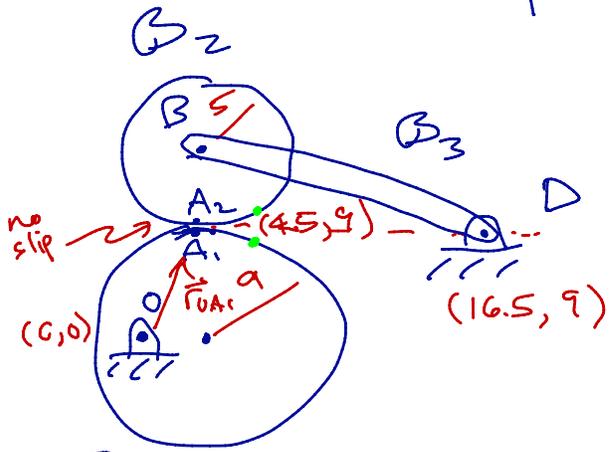
the following cam-follower setup.

Given:  $\omega_1 = 0.3 \text{ rad/s}$

Find:  $\omega_3$  ?

Solution: We know that

$$\vec{v}_{A_1} = \vec{v}_{A_2}$$



From B1:

$$\vec{v}_{A_1} = \vec{\omega}_1 \times \vec{r}_{OA_1} = (0.3 \hat{k}) \times (4.5\hat{i} + 9\hat{j})$$

$$= -2.7\hat{i} + 1.35\hat{j} = \vec{v}_{A_2}$$

From B2:

we have

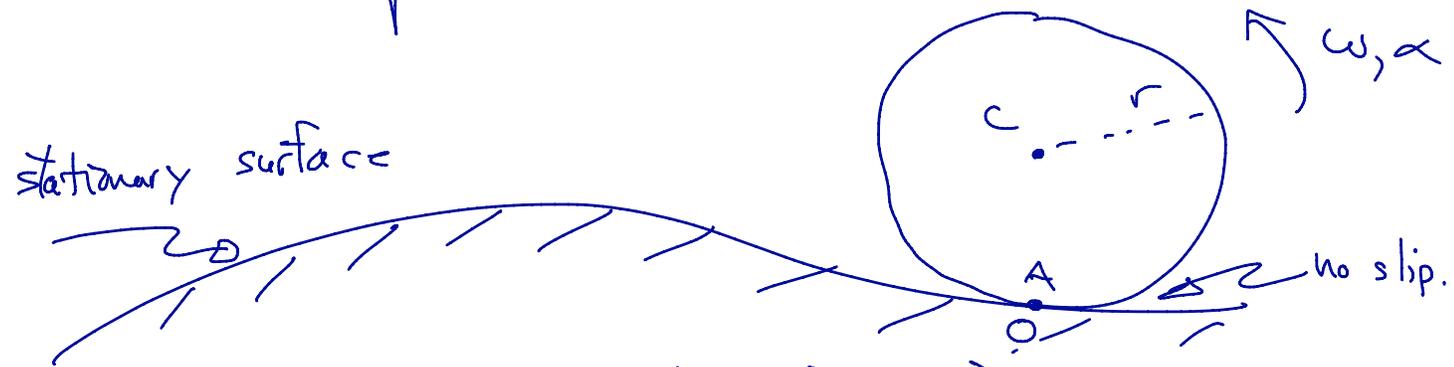
$$\vec{v}_{A_2} = \vec{v}_B + \vec{\omega}_2 \times \vec{r}_{BA_2} ; \vec{v}_B = \vec{\omega}_3 \times \vec{r}_{DB}$$

$$= \vec{\omega}_3 \times \vec{r}_{DB} + \vec{\omega}_2 \times \vec{r}_{BA_2}$$

$$\Rightarrow -2.7\hat{i} + 1.35\hat{j} = (\omega_3 \hat{k}) \times (-12\hat{i} + 5\hat{j}) + (\omega_2 \hat{k}) \times (-5\hat{j})$$

Solving:  $\omega_2 = -0.653 \text{ rad/sec}$   
 $\omega_3 = -0.113 \text{ rad/sec.}$

## No-Slip Rolling on a Curved Surface



As before we have that  $\vec{v}_O = \vec{v}_A = \vec{0}$   
 $\Rightarrow \vec{v}_C = \vec{\omega} \times \vec{r}_{AC}$

Inside of curve:

$$\vec{v}_c = \vec{\omega} \times \vec{r}_{Ac}$$

$$\vec{r}_{Ac} = r \hat{e}_n$$

$$\begin{aligned} \Rightarrow \vec{v}_c &= \vec{\omega} \times \vec{r}_{Ac} \\ &= (\omega \hat{k}) \times (r \hat{e}_n) \\ &= r\omega \hat{e}_T \end{aligned}$$

before  
 $\vec{v}_{Ac} = R\dot{\theta}$



Outside:

$$\vec{r}_{Ac} = -r \hat{e}_n$$

$$\vec{v}_c = r\omega \hat{e}_T = \vec{\omega} \times \vec{r}_{Ac}$$

