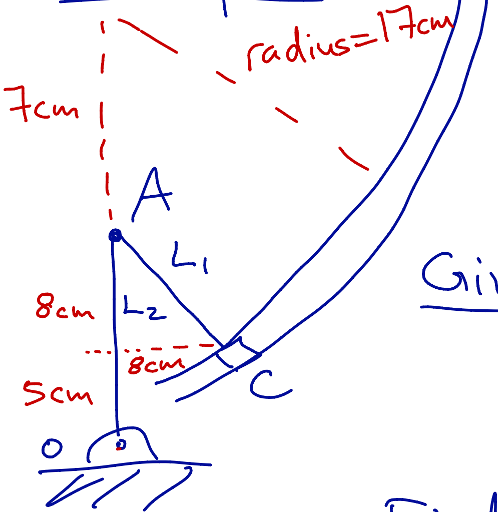
Lecture 20B: Examples

example:



Recall: link 1 denoted $L1$ has angular velocity denoted by ω_1

$L2$ has ω_2 as its ang. vel.

Given: the geometry

$\omega_1 = 3$ rad/sec $\quad \alpha_1 = -2$ rad/sec

$\rho = 17$cm

Find: α_2 of Link 2.

Relationships:

$$\vec{a}_C = \vec{a}_A + \vec{\alpha}_1 \times \vec{r}_{AC} - \omega_1^2 \vec{r}_{AC}$$

$$\vec{a}_A = \vec{\alpha}_2 \times \vec{r}_{OA} - \omega_2^2 \vec{r}_{OA}$$

geometry $\quad \vec{a}_C = \dfrac{v_C^2}{\rho} \hat{e}_n + \ddot{s}\, \hat{e}_T \qquad \ddot{s} = \dfrac{d\|\vec{v}\|}{dt}$

Approach is to find v_C and ω_2, as we'll then be able to reduce to 2 equations in the scalars α_2 and $\ddot{s}$

Step 1:

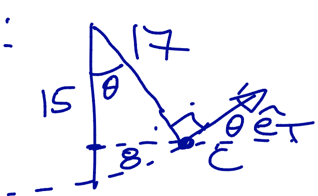
$$\vec{v}_A = \vec{\omega}_2 \times \vec{r}_{OA}$$

$$\vec{v}_C = \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{AC}$$

$$\Longrightarrow \boxed{\vec{v}_C = \vec{\omega}_2 \times \vec{r}_{OA} + \vec{\omega}_1 \times \vec{r}_{AC}}$$

We also know that $\vec{v}_C$ is really given by the scalar v_C, since its direction is constrained. $\quad \vec{v}_C = v_C\, \hat{e}_T$

Compute $\hat{e}_T$:



$$\hat{e}_T = \cos\theta\, \hat{\imath} + \sin\theta\, \hat{\jmath}$$

$$= \left(\tfrac{15}{17}\right) \hat{\imath} + \left(\tfrac{8}{17}\right) \hat{\jmath}$$

Substituting we get:

$$v_c \hat{e}_T = v_c\left(\frac{15}{17}\hat{i} + \frac{8}{17}\hat{j}\right) = (\omega_z \hat{k}) \times (13\hat{j}) + (3\hat{k}) \times (8\hat{i} - 8\hat{j})$$

Solving for v_c and ω_z: $v_c = 51$, $\omega_z = -\frac{21}{13}$.

Step 2: Sub v_c and ω_z into our initial equations above.

First let's get $\hat{e}_n$: $\hat{e}_n \cdot \hat{e}_T = 0$

$$\hat{e}_T = \left(\frac{15}{17}\right)\hat{i} + \left(\frac{8}{17}\right)\hat{j}$$

$$\hat{e}_n = \left(\frac{-8}{17}\right)\hat{i} + \left(\frac{15}{17}\right)\hat{j}$$

So doing substitution we get

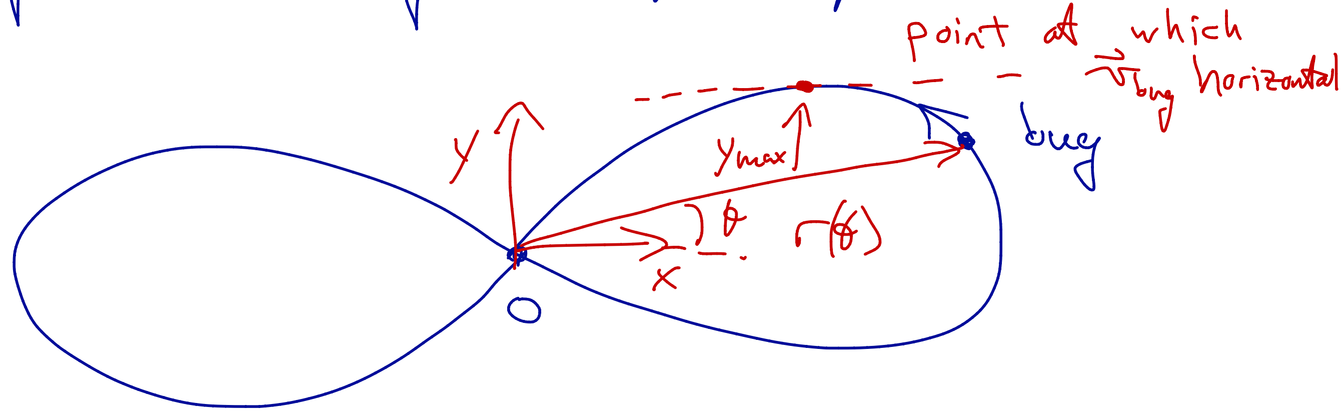
$$\frac{(51)^2}{17}\left(-\frac{8}{17}\hat{i} + \frac{15}{17}\hat{j}\right) + \ddot{s}\left(\frac{15}{17}\hat{i} + \frac{8}{17}\hat{j}\right)$$

$$= (\alpha_z \hat{k}) \times (13\hat{j}) - \left(\frac{-21}{13}\right)^2 (13\hat{j}) + (-2\hat{k}) \times (8\hat{i} - 8\hat{j}) - (3)^2(8\hat{i} - 8\hat{j})$$

2 eqns 2 unknowns

$$\boxed{\begin{array}{l} \ddot{s} = -240 \\ \alpha_z = 15.1 \text{ rad/sec}^2 \end{array}}$$

A bug moves along a figure-eight type curve



given by the polar coordinate relation

$$r = \sqrt{25\cos 2\theta}$$

and it moves at $\boxed{\text{constant speed of } 2 \text{ m/s.}}$

Determine: the acceleration vector $\vec{a}_{bug}$ at the next time the bug has purely horizontal velocity.

$$\vec{v}_{bug} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\vec{a}_{bug} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Let's find the point in question:

The velocity is purely horizontal exactly when y is maximized.

$$y = r \cdot \sin\theta \qquad \text{want } y' = 0$$

$$r = \sqrt{25\cos 2\theta}$$

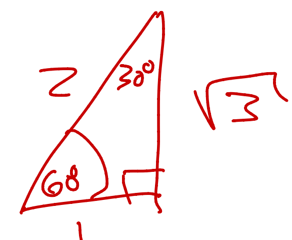
$$0 = \frac{dy}{d\theta} = r'(\theta)\sin\theta + r(\theta)\cos\theta$$

$$= \frac{1}{\cancel{2}}\frac{1}{r(\theta)}\left(-\cancel{50}\sin 2\theta\right)\sin\theta + r(\theta)\cos\theta$$

$$0 = (-25)\sin 2\theta \cdot \sin\theta + r^2(\theta)\cos\theta$$

$$0 = (-25)\sin 2\theta \sin\theta + 25\cos 2\theta \cos\theta$$

$$\Longrightarrow \quad \sin 2\theta \sin\theta = \cos 2\theta \cdot \cos\theta$$



By inspection
$$\boxed{\begin{aligned} \theta &= 30° = \pi/6 \\ r &= \sqrt{25\cos 2\theta} = \frac{5}{\sqrt{2}} \end{aligned}}$$
* See Addendum for direct solution for θ

First derivatives: $\quad \|\vec{v}_{bug}\| = 2 = \|\dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta\|$

$$\dot{r} = \frac{1}{2}\,\frac{-50\sin 2\theta}{\sqrt{25\cos 2\theta}}\cdot\dot{\theta} = \left(-\frac{\sqrt{25}\,\sin 2\theta}{\sqrt{\cos 2\theta}}\right)\cdot\dot{\theta} \longleftarrow$$

We know

$$2 = \sqrt{\dot{r}^2 + r^2\dot{\theta}^2} = \sqrt{\frac{25\sin^2 2\theta}{\cos 2\theta}\cdot\dot{\theta}^2 + 25\cos 2\theta\,\dot{\theta}^2}$$

$$= \sqrt{\frac{25}{\cos 2\theta}}\,\dot{\theta} \Longrightarrow \dot{\theta} = \frac{2\sqrt{\cos 2\theta}}{5}$$

$$\Longrightarrow \dot{r} = \frac{-5\sin 2\theta}{\sqrt{\cos 2\theta}}\cdot\dot{\theta} = -2\sin 2\theta$$

At this particular point $\dot{\theta} = \frac{\sqrt{2}}{5}$ and $\dot{r} = -\sqrt{3}$

Second derivatives:

$$\ddot{r} = \frac{d}{dt}\left\{-2\sin 2\theta\right\} = (-4\cos 2\theta)\,\dot{\theta}$$
$$= \left(-\frac{8}{5}\right)(\cos 2\theta)^{3/2}$$

$\dot{r}$ from above

$$\ddot{\theta} = \frac{d}{dt}\left\{\frac{2\sqrt{\cos 2\theta}}{5}\right\} = \frac{1}{5}\,\frac{-2\sin 2\theta}{\sqrt{\cos 2\theta}}\cdot\dot{\theta} = \left(\frac{-4}{25}\right)\sin 2\theta$$

At our point we get $\ddot{r} = \frac{-2\sqrt{2}}{5}$ and $\ddot{\theta} = \frac{-2\sqrt{3}}{25}$

Finally we can apply our formula:

$$\vec{a}_{bug} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$

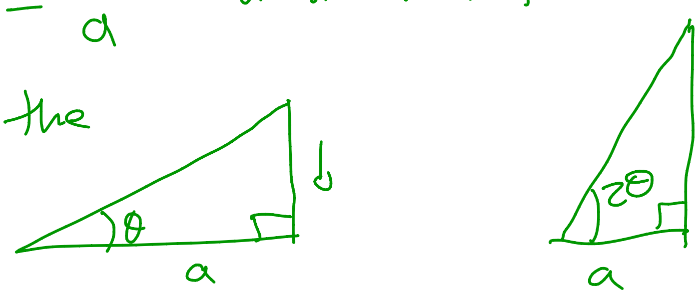
$$\Longrightarrow \quad \underline{\vec{a}_{bug} = (-0.849)\,\hat{e}_r - (1.47)\,\hat{e}_\theta} \quad m/s^2$$

$$\vec{F}_{on\,bug} = m_{bug}\,\vec{a}_{bug}.$$

* **Addendum:** to solve for θ in $\sin\theta\,\sin 2\theta = \cos\theta\,\cos 2\theta$ convert to $\tan\theta\cdot\tan 2\theta = 1$

Now let $\tan\theta = \frac{b}{a}$ and therefore $\tan 2\theta = \frac{a}{b}$

$\Longrightarrow$ this gives the triangles:



Each triangle must have hypotenuse $\sqrt{a^2+b^2}$, and are therefore similar $\Longrightarrow$ $\theta + 2\theta = 90$ and so $\theta = 30°$