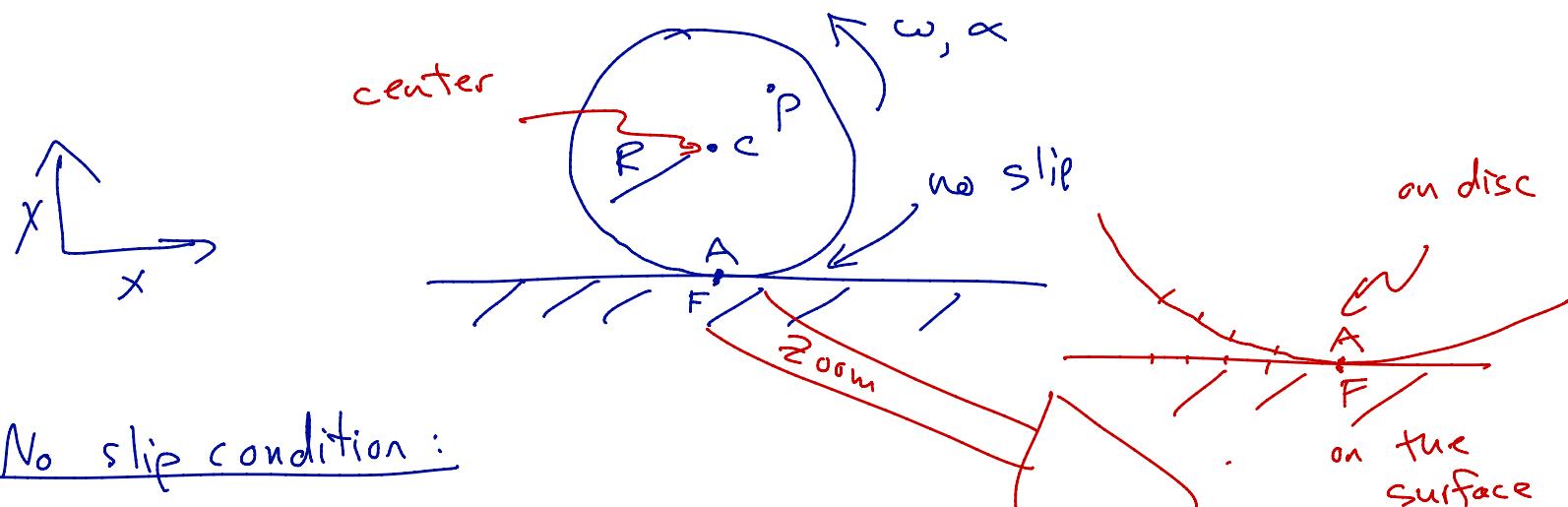


10-18-17

Lecture 20

Recap: No-slip Rolling on a Flat Surface



No slip condition:

$$\vec{\omega}_A = \vec{\omega}_F = \vec{0} \quad \text{since point } F \text{ is assumed to be stationary.}$$

Last lecture: we showed that for any point P on the disc that

$$\begin{aligned} \rightarrow \vec{\omega}_P &= \vec{\omega} \times \vec{r}_{AP} \\ \rightarrow \vec{a}_P &= \underbrace{\vec{\alpha} \times \vec{r}_{AP}} + \omega^2 (R\hat{j} - \vec{r}_{AP}) \end{aligned}$$

Point C: $\vec{\omega}_c = \vec{\omega}_A + \vec{\omega} \times \vec{r}_{AC}$

$$= (\omega \hat{k}) \times (R\hat{j}) = -R\omega \hat{i}$$

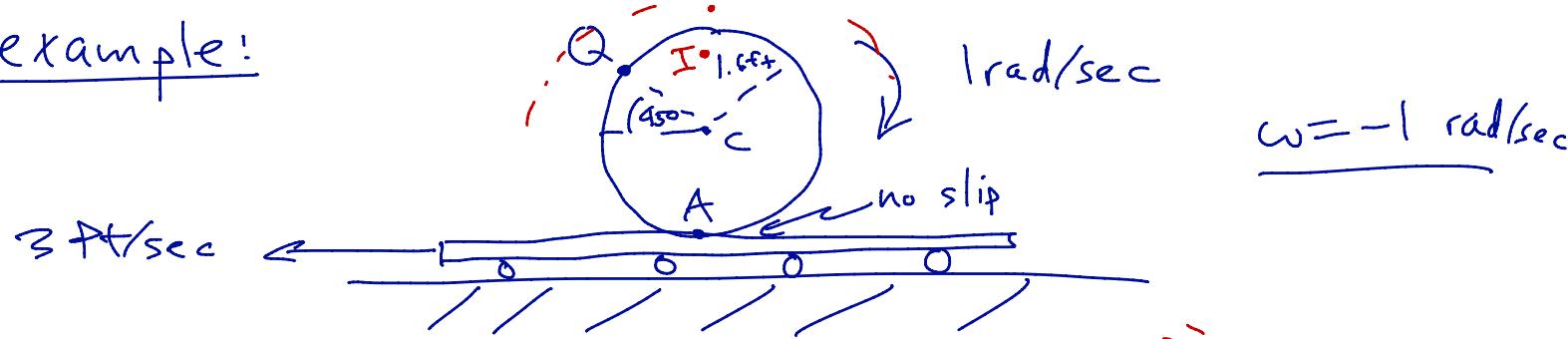


$$\vec{a}_c = \frac{d}{dt} \{ \vec{\omega}_c \} = \frac{d}{dt} \{ -R\omega(t) \hat{i} \} = -R\dot{\omega} \hat{i} = -R\alpha \hat{i}$$

Point A: $\vec{\omega}_A = \vec{\alpha}_c + \vec{\alpha} \times \vec{r}_{CA} - \omega^2 \vec{r}_{CA} = \underline{\omega^2 R\hat{j}}$

arbitrary point P: $\begin{aligned} \vec{\alpha}_P &= \vec{\alpha}_A - \vec{\alpha} \times \vec{r}_{AP} - \omega^2 \vec{r}_{AP} \\ &= \vec{\alpha} \times \vec{r}_{AP} + \omega^2 (R\hat{j} - \vec{r}_{AP}) \end{aligned}$

example:



$$\omega = -1 \text{ rad/sec}$$

- (a) determine the point I on the disc which has zero velocity.
- (b) find \vec{v}_Q .

Part (a): we know that $\vec{v}_A = -3\hat{i}$

We are looking for the point I satisfying

$$\vec{v}_I = \vec{0} = \vec{v}_A + \vec{\omega} \times \vec{r}_{AI}$$

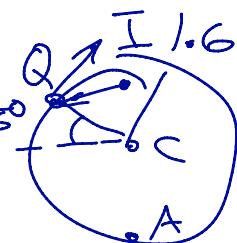
$$\vec{0} = -3\hat{i} + (-\hat{k}) \times (x\hat{i} + y\hat{j})$$

$$0 = -3\hat{i} - x\hat{j} + y\hat{i}$$

$$x=0 ; y=3\hat{j}$$

$$\vec{r}_{AI} = 3\hat{j}$$

Part (b): find \vec{v}_Q ; two ways.

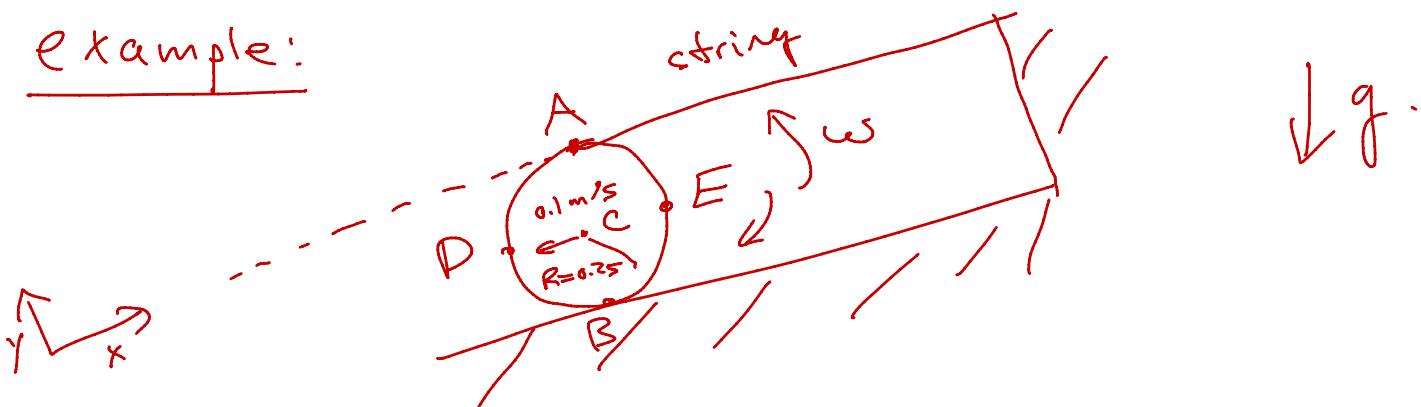


$$\textcircled{i} \quad \vec{v}_Q = \vec{v}_A + \vec{\omega} \times \vec{r}_{AQ} \quad \vec{r}_{AQ} = 1.6(-\frac{1}{\sqrt{2}}\hat{i} + (1+\frac{1}{\sqrt{2}})\hat{j})$$

$$\textcircled{ii} \quad \vec{v}_Q = \vec{\omega} \times \vec{r}_{IQ} \quad \vec{r}_{IQ} = \vec{r}_{AQ} - \vec{r}_{AI} = -1.12\hat{i} - 0.28\hat{j}$$

$$\vec{v}_Q = -0.28\hat{i} + 1.12\hat{j}$$

Example:



Given: A string is attached to the edge of the spool, and wound around it. The spool is unwinding as it slides down the decline, with its center C having speed 0.1 m/sec.

Find: the velocities at points A, B, D, E.

Solution: the key physical constraint is that $\vec{v}_A = 0$, whereas point B is allowed to slip.

We need to find ω : $\vec{v}_c = -0.1\hat{i}$

$$-0.1\hat{i} = \vec{v}_c = \vec{v}_A + \vec{\omega} \times \vec{r}_{Ac} = \vec{\omega} \times (-0.25\hat{j})$$

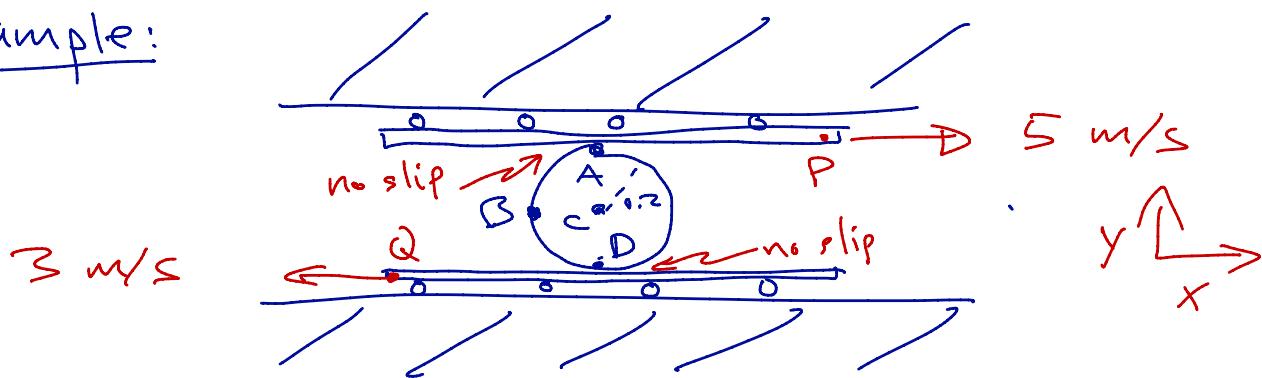
$$\Rightarrow \omega = \left(\frac{-0.1}{0.25} \right) = -0.4 \text{ rad/sec}$$

Now we can compute the velocity of any point P via $\vec{v}_P = \vec{\omega} \times \vec{r}_{AP}$

$$\vec{v}_B = \vec{\omega} \times \vec{r}_{AB} = (-0.4\hat{k}) \times (-0.5\hat{j}) = -0.2\hat{i} \text{ m/s.}$$

Other points' velocities involve similar calculations.

example:



Given: the picture, and there is no slip surfaces.
(radius = 0.2 m)

Find: the velocities of points B and C.

No slip constraint: $\vec{v}_A = 5\hat{i}$
 $\vec{v}_D = -3\hat{i}$

Let's get ω : $\vec{v}_A = \vec{v}_D + \vec{\omega} \times \vec{r}_{DA}$
 $5\hat{i} = -3\hat{i} + (\omega\hat{k}) \times (0.4\hat{j})$
 $\Rightarrow 8\hat{i} = -\omega(0.4)\hat{i} \Rightarrow \omega = -20 \text{ rad/sec.}$

\vec{v}_B : $\vec{v}_B = \vec{v}_D + \vec{\omega} \times \vec{r}_{DB}$
 $= -3\hat{i} + (-20\hat{k}) \times (-0.2\hat{i} + 0.2\hat{j})$
 $= \hat{i} + 4\hat{j}$

\vec{v}_C : $\vec{v}_C = 1. \hat{i} \text{ m/s} \left\{ \begin{array}{l} = \vec{v}_B + \vec{\omega} \times \vec{r}_{DC} \\ = \vec{v}_A + \vec{\omega} \times \vec{r}_{AC} \end{array} \right\} \Rightarrow$

example:

