

10-16-17

Lecture #19

Robotics: seminar today 4pm
2AOS Seibel

Rigid Bodies: Instantaneous Center of Zero Velocity

$$\vec{v}_A = \vec{v}_c + \vec{\omega} \times \vec{r}_{cA}$$

Can we always satisfy the equation

$$\vec{0} = \vec{v}_c + \vec{\omega} \times \vec{r}_{cA}$$

for some point A ??

$$\Rightarrow -\vec{v}_c = (\vec{\omega} \hat{k}) \times (x \hat{i} + y \hat{j})$$

$$-v_c^x \hat{i} - v_c^y \hat{j} = -\omega y \hat{i} + \omega x \hat{j} \quad \text{for } x \neq y$$

Question: Can we solve this equation given \vec{v}_c ??

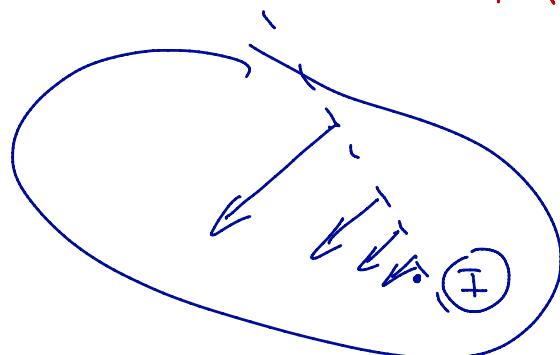
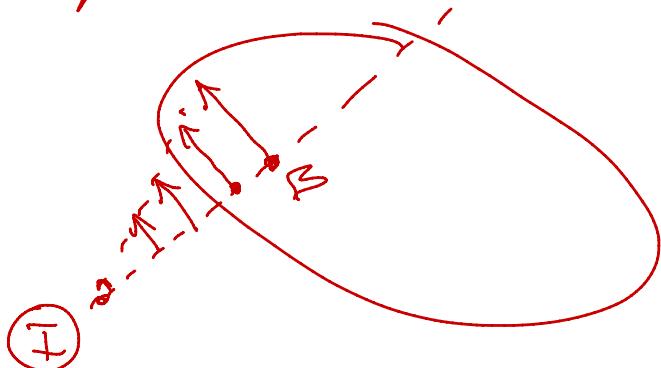
Yes! ~~if~~ provided that $\omega \neq 0$.

Note: the point may be on a virtual part of body.
(i.e., outside rigid body).

This point A is called the Instantaneous Center of Zero Velocity.

Frequently denoted by \textcircled{I} .

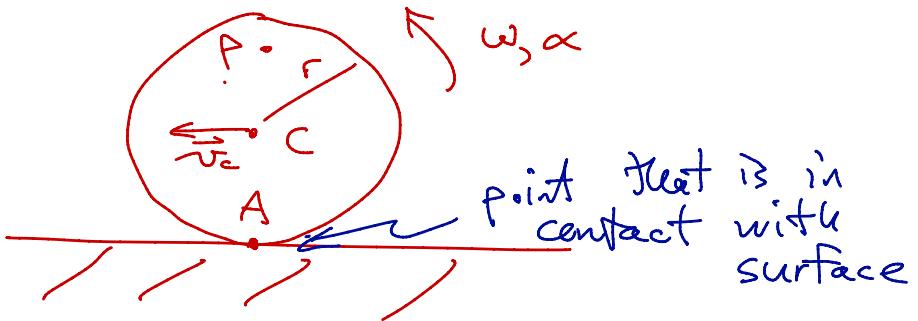
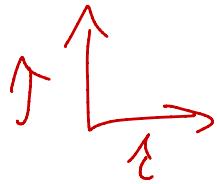
$$\vec{v}_B = \vec{\omega} \times \underbrace{\vec{r}_{IB}}_{r_{IB}}$$



Two velocities given



Rolling: No slip:



No slip condition: $\vec{v}_A = 0$

$$\vec{v}_c = \vec{v}_A + \vec{\omega} \times \vec{r}_{Ac} = (\omega \hat{i}) \times (r \hat{j}) = -\omega r \hat{i}$$

$$\vec{a}_c = \frac{d}{dt} \{ \vec{v}_c \} = \frac{d}{dt} \{ -\omega(t) r \hat{i} \} = -\dot{\omega} r \hat{i} = -r \alpha \hat{i}$$

What about the acceleration at point A ?

$$\vec{a}_A = \vec{a}_c + \vec{\alpha} \times \vec{r}_{cA} - \omega^2 \vec{r}_{cA}$$

$$= -r \alpha \hat{i} + (\alpha \hat{i}) \times (-r \hat{j}) - \omega^2 (-r \hat{j})$$

$$= -r \alpha \hat{i} + r \alpha \hat{k} + \omega^2 r \hat{j} = \underline{\underline{\omega^2 r \hat{j}}}$$

independent of α !!

Looking at accelerations on the body:

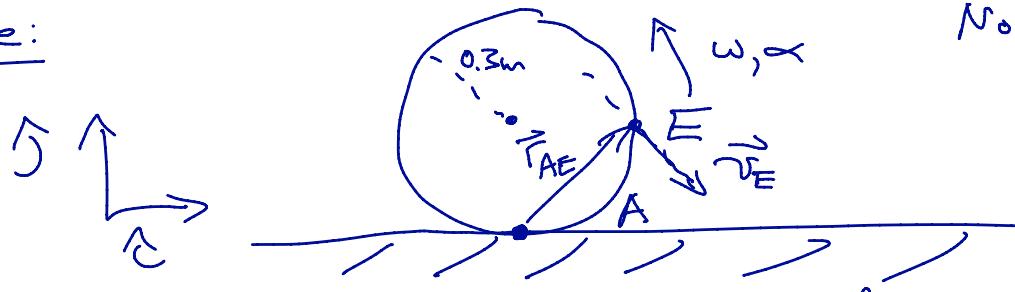
$$\vec{v}_P = \vec{\omega} \times \vec{r}_{AP}$$

$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{AP} - \omega^2 \vec{r}_{AP}$$

$$= \vec{\alpha} \times \vec{r}_{AP} + \omega^2 (r \hat{j} - \vec{r}_{AP})$$



Example:



No slip rolling

The cylinder rolls without slipping and

$$\omega = -2 \text{ rad/sec}$$

$$\alpha = 1.5 \text{ rad/sec}^2$$

Find: \vec{v}_E and \vec{a}_E

Velocity: $\vec{v}_E = \vec{\omega} \times \vec{r}_{AE} = (-2\hat{k}) \times (0.3\hat{i} + 0.3\hat{j})$

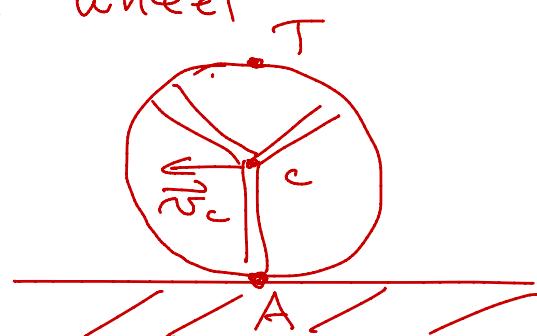
$$= 0.6\hat{i} - 0.6\hat{j} \text{ m/s}$$

Acceleration: formula we derived says $\vec{a}_E = \vec{\alpha} \times \vec{r}_{AE} + \vec{\omega}^2 \vec{r}_{AE}$

$$\vec{a}_E = (1.5\hat{k}) \times (0.3\hat{i} + 0.3\hat{j}) + (-2)^2 (0.3\hat{j} - (0.3\hat{i} + 0.3\hat{j}))$$

$$= -1.65\hat{i} + 0.45\hat{j} \text{ m/s}^2$$

Example: bicycle wheel



$$||\vec{v}_c|| = 20 \text{ mph}$$

$x \hat{y} \hat{x}$

What's the velocity at point C?

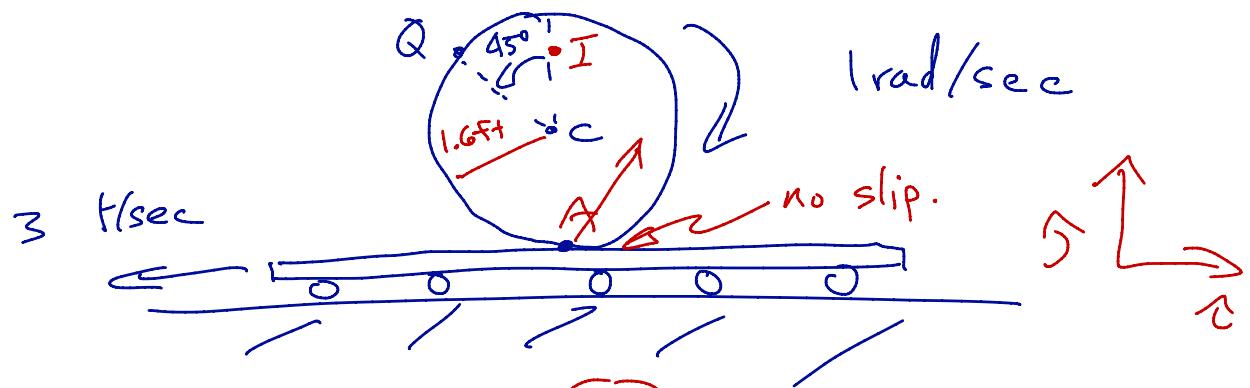
$$\vec{v}_C = -20\hat{i} = \vec{\omega} \times \vec{r}_{AC} = \omega \hat{k} \times \hat{j}$$

$$\vec{v}_T = \vec{\omega} \times \vec{r}_{AT} = \vec{\omega} \times (2\vec{r}_{AC}) = 2\vec{\omega} \times \vec{r}_{AC} = 2\vec{v}_C$$

$$= -40\hat{i} \text{ mph.}$$

Also $\omega = \frac{v_0}{2r} = \frac{20}{r}$

example:



(a) determine the point I

(b) find \vec{r}_Q .

Part a: know that $\vec{v}_A = -3\hat{i}$

Seek the point satisfying $\vec{v}_I = \vec{v}$

Set $\vec{r}_{AI} = a\hat{i} + b\hat{j}$ and by definition

$$\vec{v} = \vec{\omega} + \vec{\omega} \times \vec{r}_{AI} = -3\hat{i} + (- :) a\hat{i} + b\hat{j}$$

$$\vec{v} = -3\hat{i} - a\hat{j} + b\hat{i} \Rightarrow a = -3 \quad \text{---} \quad \boxed{a = -3}$$

$$\Rightarrow \vec{r}_{AI} = 3\hat{j}$$