10-11-17

Lecture #17

So far our focus has been on <u>kinematics</u>: study of motion based on space and time.

<u>Kinetics</u>: the study of motion taking external influences (e.g., forces) into consideration. In order to do this we need to bring masses and forces into the picture, additionally.

Newton's laws of motion: (basis for classical mechanics).

<u>first law</u>: if there is no <u>net</u> force of a particle then its velocity is constant:

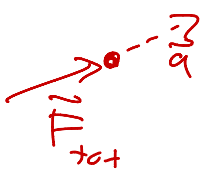
choices: (a) $\vec{v} \equiv 0$

(a') $\vec{v} \equiv$ const.

<u>second law</u>: the acceleration of a particle is equal to $\frac{1}{m}\vec{F}_{tot}$, where $\vec{F}_{tot}$ is the sum of all forces acting on it.

Assumes a point particle/mass

$$\vec{F}_{tot} = m\vec{a} = \frac{d}{dt}\{\underbrace{\vec{p}}_{m\vec{v}\text{ momentum}}\}$$



<u>third law</u>: when particle #1 exerts force $\vec{F}_{12}$ on particle #2, then particle #2 exerts $\vec{F}_{21} = -\vec{F}_{12}$ on particle #1

In the Big Picture there are two basic ways we apply $\vec{F} = m\vec{a}$:

1. <u>method of assumed forces</u>: know forces $\Rightarrow$ determine motion
2. <u>method of assumed motion</u>: assume time paths $\Rightarrow$ compute forces.

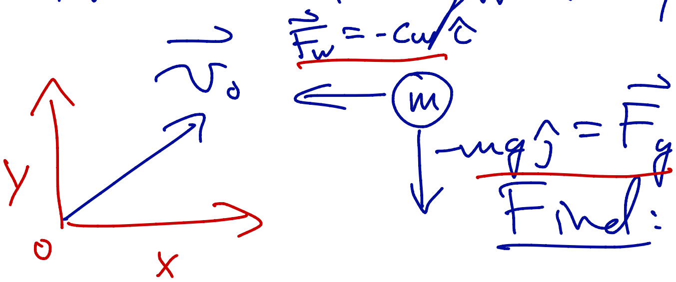
example: "assumed forces"

A cannonball of mass m is shot from the origin with some initial velocity

$$v_0 = v_{0x}\,\hat{i} + v_{0y}\,\hat{j}$$

The forces that act on it are constant in time and given by: (i) gravity $\vec{F}_g = -mg\,\hat{j}$

(ii) wind force $= -c_w\,\hat{i}$



Find: the time-parametrized trajectory. $\vec{r}(t)$

$$m\vec{a} = \vec{F}_{tot} = \vec{F}_w + \vec{F}_g$$

$$\ddot{\vec{r}}(t) = \vec{a} = \left(-\frac{c_w}{m}\right)\hat{i} + (-g)\,\hat{j} = \vec{a}_0 \Leftarrow \text{constant.}$$

$$\dot{\vec{r}}(t) = \vec{v} = \vec{v}_0 + \int_0^t \vec{a}_0\,dt = \vec{v}_0 + \vec{a}_0 \cdot t$$

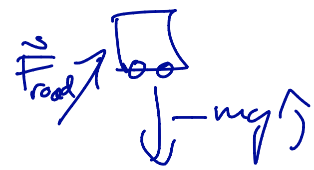
$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t)\,dt \implies \vec{r}(t) = \left(v_{0x}t - \frac{c_w}{2m}t^2\right)\hat{i} + \left(v_{0y}t - \frac{g}{2}t^2\right)\hat{j}$$

example: "method of assumed motion"

A car of mass m drives on a road with a sinusoidal elevation profile, with a constant horizontal speed. Road profile is $y = A\sin(kx)$. What is the force on the car as it travels? Find $\vec{F}_{road}$.

Want to find $\vec{a}(t) = \frac{1}{m}\vec{F}_{tot}$

$$\Longrightarrow m\vec{a}(t) + mg\hat{j} = \vec{F}_{road}$$



① express the motion as

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = x_0 + v_0 t$$
$$y(t) = A\sin(kx(t))$$

② find $\vec{a}(t)$:

$$\Longrightarrow \vec{v}(t) = \frac{d}{dt}\left\{(x_0+v_0t)\hat{i} + A\sin(k(x_0+v_0t))\hat{j}\right\}$$

$$= v_0\hat{i} + A\cos(k(x_0+v_0t))kv_0\hat{j}$$

differentiate again

$$\vec{a}(t) = -A(kv_0)^2\sin(k(x_0+v_0t))\hat{j}$$

③ forces acting on car

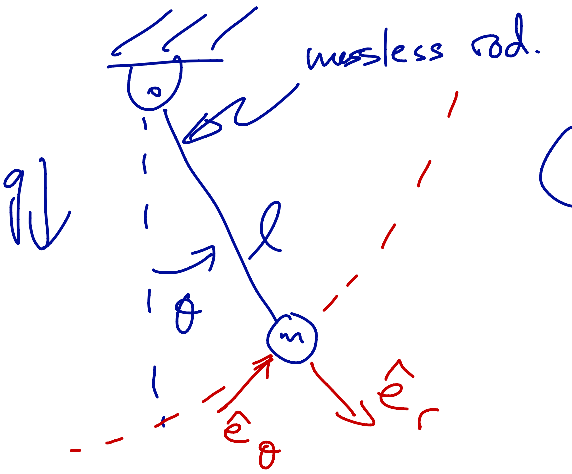
$$m\vec{a} = \vec{F}_{tot} = \vec{F}_g + \vec{F}_{road}$$

$$\Longrightarrow \vec{F}_{road} = mg - mA(kv_0)^2\sin(k(x_0+v_0t))\hat{j}$$

has the form $c(t)\hat{j}$

force supplied to generate the motion.

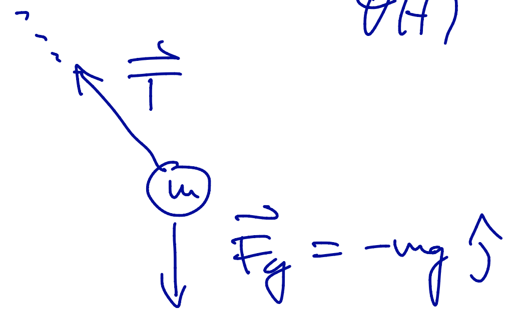
example: revisit the simple pendulum



Find: equation of motion. $\theta(t)$

(1) FBD

$$\vec{T} = -T\,\hat{e}_r$$
$$\vec{F}_g = -mg\,\hat{\jmath}$$



(2) ~~acceleration in polar coords:~~

$$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\cancel{\dot{r}}\dot{\theta})\hat{e}_\theta = \underline{-\ell\dot{\theta}^2\hat{e}_r + \ell\ddot{\theta}\hat{e}_\theta}$$

(3) Newton's second law:

$$\underline{m\vec{a}} = \vec{T} + \vec{F}_g = \underline{-T\hat{e}_r - mg\,\hat{\jmath}}$$

(4) Algebra: $\hat{\jmath} = -\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta$ sub in above.

$$m(-\ell\dot{\theta}^2\hat{e}_r + \ell\ddot{\theta}\hat{e}_\theta) = -T\hat{e}_r - mg(-\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta)$$

$\hat{e}_\theta$: $m\ell\ddot{\theta} = -mg\sin\theta$

$$\Rightarrow \underline{\ddot{\theta} = -(g/\ell)\sin\theta}$$

$\hat{e}_r$: gives T.

for small θ we can approximate to get $\ddot{\theta} \simeq -(g/\ell)\theta$ which is a DE that has an elementary solution.