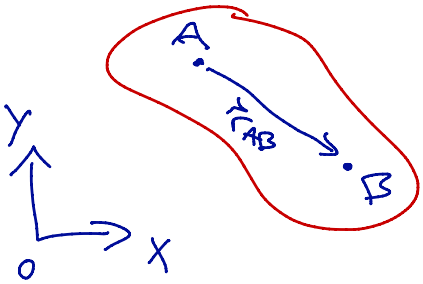
10-09-17

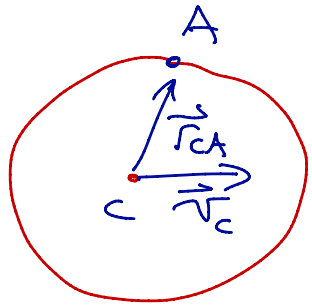
Lecture #16

Important Rigid Body Formulas



$$\boxed{\begin{aligned}\vec{v}_B &= \vec{v}_A + \vec{\omega} \times \vec{r}_{AB}\\ \vec{a}_B &= \vec{a}_A + \vec{\alpha} \times \vec{r}_{AB} - \omega^2 \vec{r}_{AB}\end{aligned}}$$

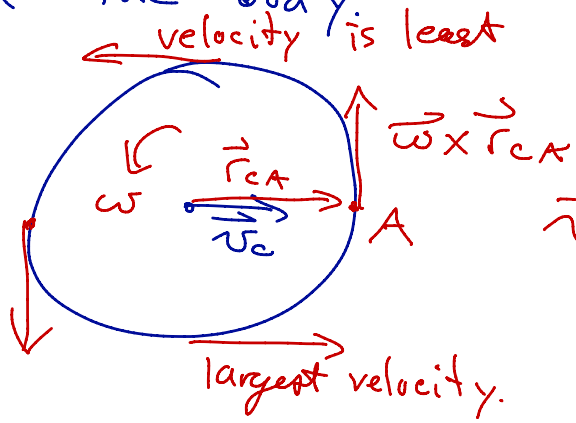
example:



$$\vec{\omega} = \omega \hat{k}$$
$$\vec{\alpha} = \alpha \hat{k}$$

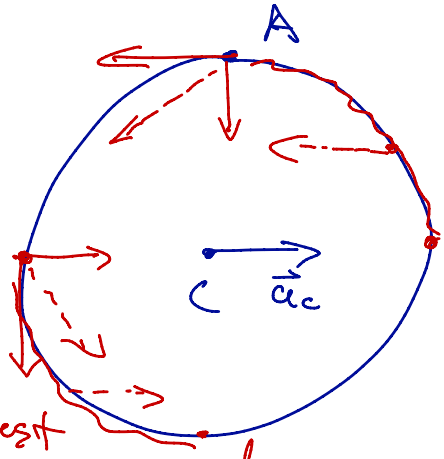
If we know $\vec{v}_C$ we can compute the velocity anywhere on the body.

velocity:



$$\vec{v}_A = \vec{v}_C + \vec{\omega} \times \vec{r}_{CA}$$

acceleration:

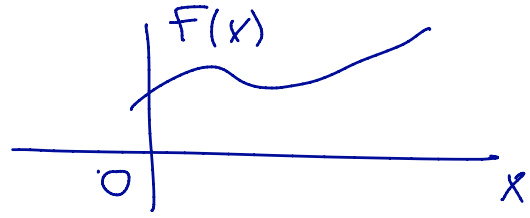


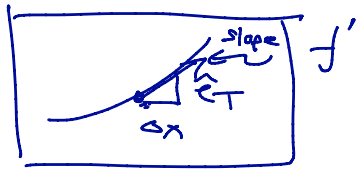
largest somewhere in III rd quadrant

$$\vec{a}_A = \vec{a}_C + \underbrace{\vec{\alpha} \times \vec{r}_{CA} - \|\omega\|^2 \vec{r}_{CA}}$$

Recall the following formula:

$$k = \frac{1}{\rho} = \frac{|f''(x)|}{(1+f'(x)^2)^{3/2}}$$



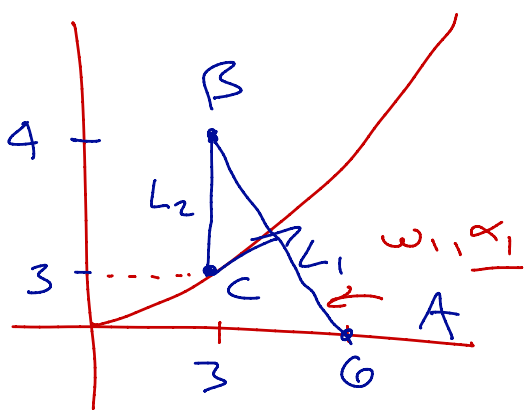


$\Delta y = f'\Delta x$

$\vec{v} = \hat{\imath} + f'\hat{\jmath}$

example:

$y = \frac{x^2}{3}$



Last lecture: we determined $\vec{v}_c$ given ω_1

Today: given both ω_1 and α_1

find: $\vec{a}_c$

$$\vec{a}_B = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha}_1 \times \vec{r}_{AB} - \omega_1^2 \vec{r}_{AB}$$

$$\vec{a}_c = \vec{a}_B + \vec{\alpha}_2 \times \vec{r}_{BC} - \omega_2^2 \vec{r}_{BC}$$

2 scalar equation

3 scalar unknown. $\Longrightarrow$ $\vec{a}_c = \vec{\alpha}_1 \times \vec{r}_{AB} - \omega_1^2 \vec{r}_{AB} + \vec{\alpha}_2 \times \vec{r}_{BC} - \omega_2^2 \vec{r}_{BC}$

from last lecture

Need some other relationship between the vars:

$$\longrightarrow \vec{a}_c = \ddot{s}\hat{e}_T + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

$|\dot{s}| = \|\vec{v}_c\|$

We know from last lecture: $\hat{e}_T = \frac{1}{\sqrt{5}}(\hat{\imath} + 2\hat{\jmath})$

$\hat{e}_n \cdot \hat{e}_T = 0$, points in: $\hat{e}_n = -\frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{\jmath} = \frac{1}{\sqrt{5}}(-2\hat{\imath} + \hat{\jmath})$

apply the curvature formula: $k = \frac{1}{\rho} = \frac{|f''(x)|}{(1+f'(x)^2)^{3/2}} = \frac{\frac{2}{3}}{\left(1+\left(\frac{2}{3}\cdot 3\right)^2\right)^{3/2}}$

$$= \frac{2}{3\cdot 5^{3/2}}$$

Substituting back into the $\vec{a}_c$ expression:

$$\vec{a}_c = \frac{\ddot{s}}{\sqrt{5}}(\hat{\imath} + 2\hat{\jmath}) + \left(\frac{2}{3\cdot 5^{3/2}}\right)\overbrace{\left(\tfrac{3}{2}\omega_1\sqrt{5}\right)}^{v_c}{}^2\left(\frac{1}{\sqrt{5}}(-2\hat{\imath} + \hat{\jmath})\right)$$

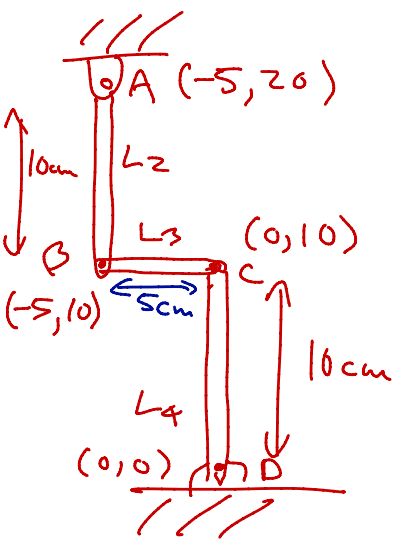
$$= (\alpha_1 \hat{k}) \times (-3\hat{\imath} + 4\hat{\jmath}) - \omega_1^2(-3\hat{\imath} + 4\hat{\jmath}) + (\alpha_2 \hat{k}) \times (-\hat{\jmath}) - \omega_2^2(-\hat{\jmath})$$

$\Longrightarrow$ solve for $\ddot{s}$ and α_2

General Gameplan:

- write down the velocity and acceleration relationships between the points of interest.
- use the rigid body couplings and geometric constraints to reduce the number of variables.
- solve the remaining equations.

example:



Given: Link 4 has

$\omega_4 = 2$ rad/sec

$\alpha_4 = 0$ rad/sec^2

Determine: ω_3, α_3

Velocities:

$$\vec{v}_C = \vec{\omega}_4 \times \vec{r}_{DC}$$

$$v_B^x \hat{i} + v_B^y \hat{j} = \vec{v}_B = \vec{v}_C + \underbrace{\vec{\omega}_3}_{\omega_3 \hat{k}} \times \vec{r}_{CB}$$

2 scalar eqns

3 scalar unknowns

2 unknowns