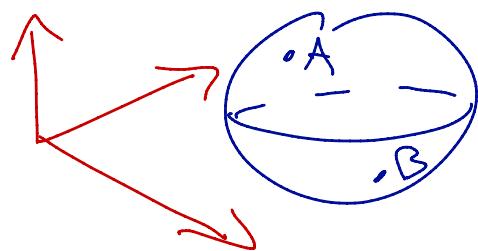


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Lecture # 15

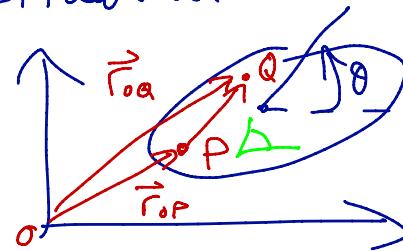
- Reminder:
- Quiz #2 starts tomorrow
 - no Friday lecture
 - recorded worked examples: available tomorrow.

Motion of Rigid Bodies



$$\|\vec{r}_A(t) - \vec{r}_B(t)\| = \text{const}$$

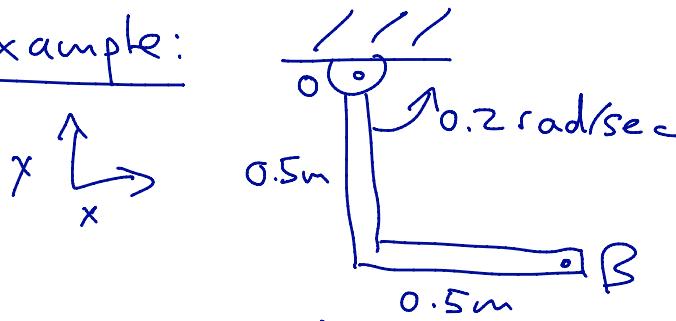
2D situation



$$\begin{aligned}\vec{\omega} &= \dot{\theta} \hat{k} \\ &= \omega \hat{k}\end{aligned}$$

$$\boxed{\vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}}$$

example:



Find: \vec{v}_B

know: $\vec{\omega} = 0.2 \hat{k}$

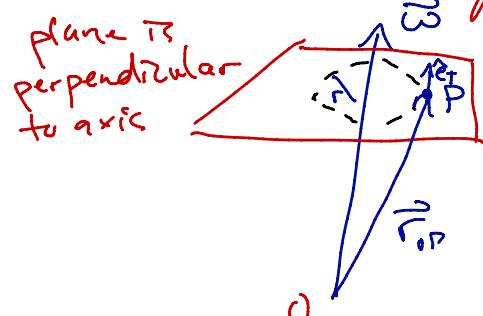
$$\vec{r}_{OB} = 0.5\hat{i} - 0.5\hat{j}$$

$$\Rightarrow \vec{v}_B = \vec{v}_O + \vec{\omega} \times \vec{r}_{OB} = 0 + (0.2 \hat{k}) \times (0.5\hat{i} - 0.5\hat{j}) \\ = 0.1 \hat{k} \times (\hat{i} - \hat{j}) = 0.1(\hat{j} + \hat{i}) \text{ m/s}$$

More on Angular Velocity with the 3D Picture in Mind

~~$\vec{\omega}$~~ $\vec{\omega}$ = in 3D our angular velocity vector can point in any direction.

$\|\vec{\omega}\|$ = angular speed. $\vec{\omega} = \alpha \times \vec{B}$ of rotation

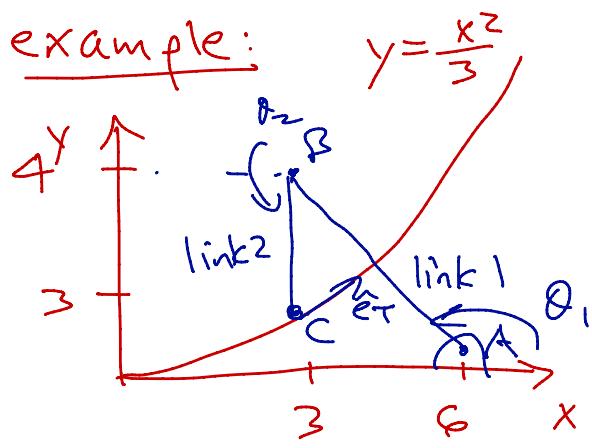


$$\vec{v} = r \|\vec{\omega}\| \hat{e}_T$$

$$\hat{e}_T = \hat{\omega} \times \hat{r}_{OP}$$

$$r = \|\vec{r}_{OP}\| \sin \theta$$

$$\Rightarrow \vec{v} = \vec{\omega} \times \vec{r}_{OP}$$



Point C is constrained to move in the parabolic slot.

Given: $\dot{\theta}_1 = \omega_1$, and the locations of A, B and C.

Find: \vec{v}_c

$$\vec{v}_c = \vec{v}_B + \vec{\omega}_2 \times \vec{r}_{BC}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{AB}$$

$$\vec{v}_c = \vec{\omega}_1 \times \vec{r}_{AB} + \vec{\omega}_2 \times \vec{r}_{BC}$$

$$v_c^x \hat{i} + v_c^y \hat{j} = \vec{\omega}_1 \times \vec{r}_{AB} + \vec{\omega}_2 \times \vec{r}_{BC}$$

Add this stage: 2 scalar eqns

3 scalar unknowns.

We know that $\vec{v}_c = v_c \hat{e}_T$

$$f(x) = \frac{x^2}{3}; f'(x) = \frac{2}{3}x$$

$$f'(3) = 2 \Rightarrow \hat{e}_T = \frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})$$

slope normalized

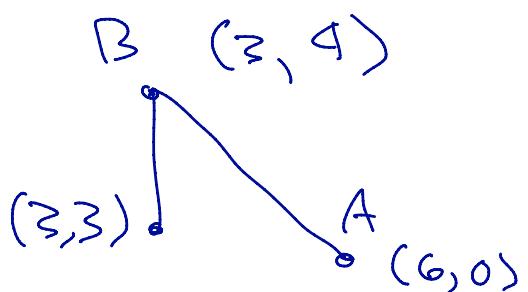
$$= \left(\frac{v_c}{\sqrt{5}} \right) (\hat{i} + 2\hat{j})$$

2 scalar unknowns.

$$\vec{v}_c \hat{e}_T = (\omega_1 \hat{i}) \times \vec{r}_{AB} + (\omega_2 \hat{i}) \times \vec{r}_{BC}$$

geometry:

$$\begin{aligned} \vec{r}_{AB} &= -3\hat{i} + 4\hat{j} \\ \vec{r}_{BC} &= -\hat{j} \end{aligned}$$



$$\Rightarrow \frac{\vec{v}_c}{\sqrt{5}} (\hat{i} + 2\hat{j}) = (\omega_1 \hat{i}) \times (-3\hat{i} + 4\hat{j}) + (\omega_2 \hat{i}) \times (-\hat{j})$$

$$\frac{\vec{v}_c}{\sqrt{5}} (\hat{i} + 2\hat{j}) = (-\omega_1 4 + \omega_2) \hat{i} - 3\omega_1 \hat{j}$$

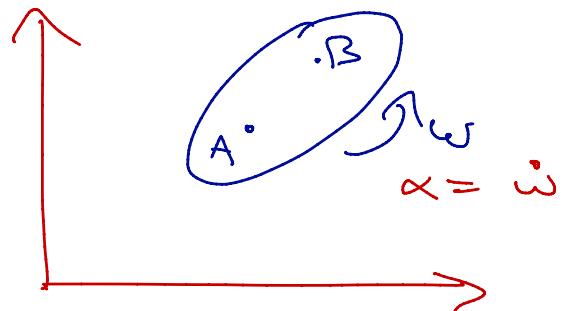
Equating components:

$$\begin{aligned} \frac{\vec{v}_c}{\sqrt{5}} &= -4\omega_1 + \omega_2 \Rightarrow v_c = -\frac{3\sqrt{5}}{2}\omega_1 \\ \frac{2\vec{v}_c}{\sqrt{5}} &= -3\omega_1 \end{aligned}$$

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Accelerations on Rigid Bodies

$$\begin{aligned}\vec{r}_{AB} &= \vec{r}_A + \vec{r}_{AB} \\ \vec{v}_B &= \vec{v}_A + \underbrace{\vec{\omega} \times \vec{r}_{AB}}_{\vec{v}_{AB}}\end{aligned}$$



$$\vec{a}_B = \frac{d}{dt} \{ \vec{v}_B \} = \vec{a}_A + \frac{d}{dt} \{ \vec{\omega} \times \vec{r}_{AB} \}$$

$$= \vec{a}_A + \vec{\omega} \times \vec{r}_{AB} + \vec{\omega} \times \vec{r}_{AB}$$

$$= \vec{a}_A + \vec{\alpha} \times \vec{r}_{AB} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{AB})$$

$\uparrow \alpha \hat{r}$

$$\boxed{\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{AB} - \|\vec{\omega}\|^2 \vec{r}_{AB}}$$

$\vec{\omega} \times \vec{r}_{AB}$
 $\vec{\omega} \times (\vec{\omega} \times \vec{r}_{AB})$
 -ve wrt \vec{r}_{AB}