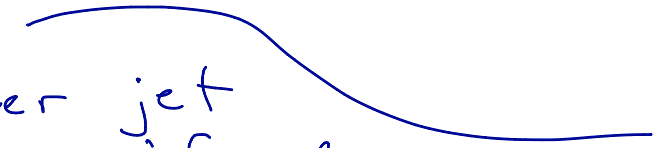
10/02/17

Lecture #14

① Suppose you see the vapor trail of a passenger jet in the sky, and you are informed that it's flying at a constant speed (500mph). Which of the following can you determine?



(a) $\vec{r}^{\,d}_{Jet}(s)$

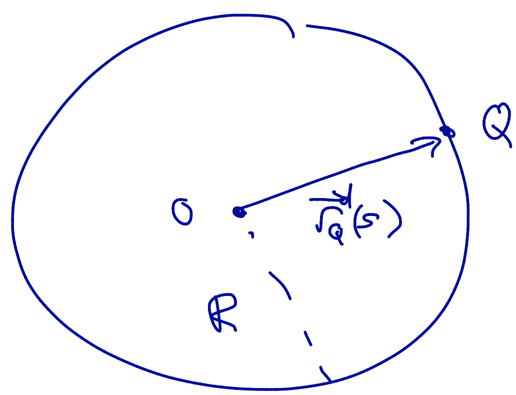
$s(t) = 500 \cdot t + \text{const}$

(b) $\vec{r}_{Jet}(t) = \vec{r}^{\,d}_{Jet}(s(t))$

(c) both (a) & (b) $\boxed{\checkmark}$

(d) none of the above

② Consider the circle



Which of the following is true?

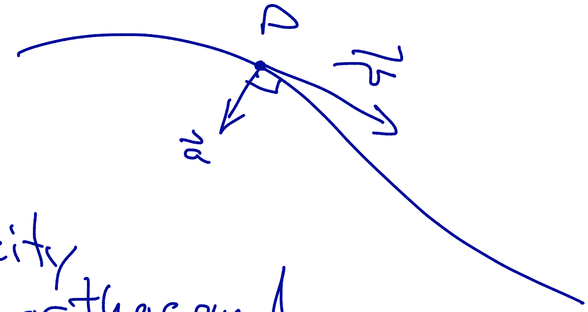
(a) $\|\vec{r}^{\,d\,''}_{Q}(s)\| = R$

(b) $\|\vec{r}^{\,d\,''}_{Q}(s)\| = \frac{1}{R}$ $\boxed{\checkmark}$

(c) $\|\vec{r}^{\,d\,''}_{Q}(s)\| = 1$

(d) none of these.

3



Suppose that particle P is moving along the shown curve and that the velocity and acceleration vectors are orthogonal. Which of the following is true:

$\hat{e}_n \perp \hat{e}_T$

a. the particle is speeding up.

b. slowing down

$\vec{a} = \ddot{s}\,\hat{e}_T + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$

c. neither a nor b ✓

d. insufficient information.

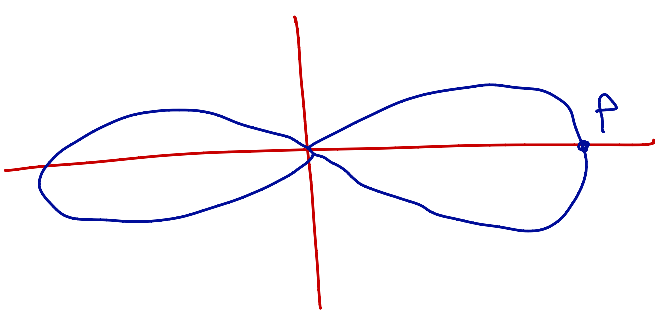
Major formulas:

$$\vec{v}_P = \dot{s}(t)\,\hat{e}_T(t)$$

$$\vec{a}_P = \ddot{s}\,\hat{e}_T + \frac{\dot{s}^2}{\rho}\,\hat{e}_n$$

example: Consider the path defined in polar coords

$r^2(\theta) = 2\cos 2\theta \ \text{ft}^2$ (Bernoulli lemniscate)



Given: at time $t=0$ a cyclist is at point P,

$$\boxed{\theta(t) = 3t^2 + 2t}$$

Find: $\vec{v}_P$, $\vec{a}_P$ and ρ at point/time P.

We have two sets of equivalent formulae:

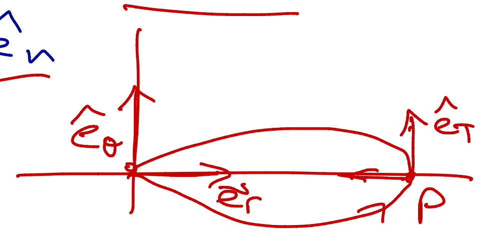
$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \quad ; \quad \vec{a} = (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + (\ddot{r} - r\dot{\theta}^2)\hat{e}_r$$

$$\vec{v} = \dot{s}\,\hat{e}_T \qquad \vec{a} = \ddot{s}\,\hat{e}_T + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

$$\dot{s} = r\dot{\theta}$$

$$\hat{e}_T = \hat{e}_\theta$$

$$\hat{e}_n = -\hat{e}_r$$



$$\frac{\dot{s}^2}{\rho} = r\dot{\theta}^2 - \ddot{r} \Longrightarrow \rho = \frac{\dot{s}^2}{r\dot{\theta}^2 - \ddot{r}}$$

To complete the problem we need $\dot{\theta}, \ddot{\theta}, r, \dot{r}, \ddot{r}$ at $t=0, \theta=0$

At point P we have $r^2 = 2\cos 2\theta \Longrightarrow r = \sqrt{2}$

$\dot{\theta} = 6t + 2, \quad \ddot{\theta} = 6$

To get the derivatives of r:

$$r^2 = 2\cos 2\theta$$

$$2r\dot{r} = (-4\sin 2\theta)\dot{\theta} \Longrightarrow \dot{r}(0) = 0$$

$$2\dot{r}^2 + 2r\ddot{r} = (-2\cos 2\theta)(2\dot{\theta})^2 - 4\sin(2\theta)\ddot{\theta}$$

$$\Longrightarrow \ddot{r}(0) = -8\sqrt{2}$$

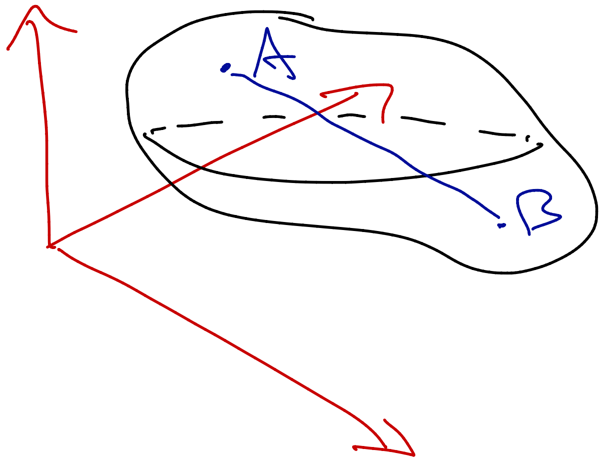
Now apply the relationships: $\vec{v} = r\dot{\theta}\hat{e}_\theta = 2\sqrt{2}\,\hat{j}$

$\|\vec{v}\| = \dot{s} = 2\sqrt{2}$

$$\rho = \frac{\dot{s}^2}{r\dot{\theta}^2 - \ddot{r}} = \frac{\sqrt{2}}{5}$$

$$\vec{a} = (r\ddot{\theta})\hat{j} - \frac{\dot{s}^2}{\rho}\hat{i}$$

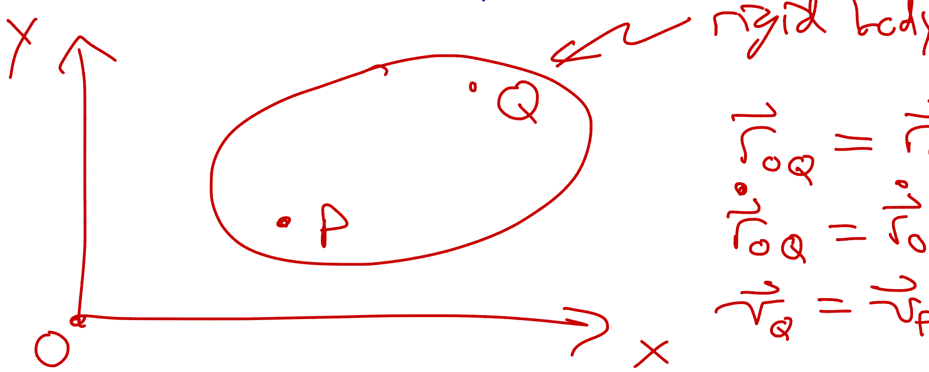
Motion of Rigid Bodies



A $\boxed{\text{rigid body}}$: an object where the distance between two points attached to it satisfy

$$\|\vec{r}_A(t) - \vec{r}_B(t)\| = \text{const}$$

If the "z-coordinate" of every point on the body is constant, we say we have planar motion.

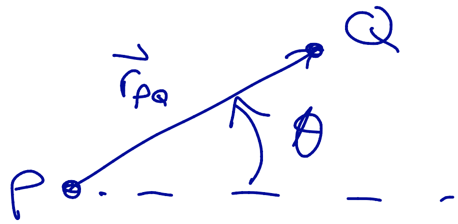


$$\vec{r}_{OQ} = \vec{r}_{OP} + \vec{r}_{PQ}$$

$$\dot{\vec{r}}_{OQ} = \dot{\vec{r}}_{OP} + \dot{\vec{r}}_{PQ}$$

$$\vec{v}_Q = \vec{v}_P + \dot{\vec{r}}_{PQ}$$

examine



So in polar coordinates:

$$\vec{r}_{PQ} = \|\vec{r}_{PQ}\| (\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$ due rigidity

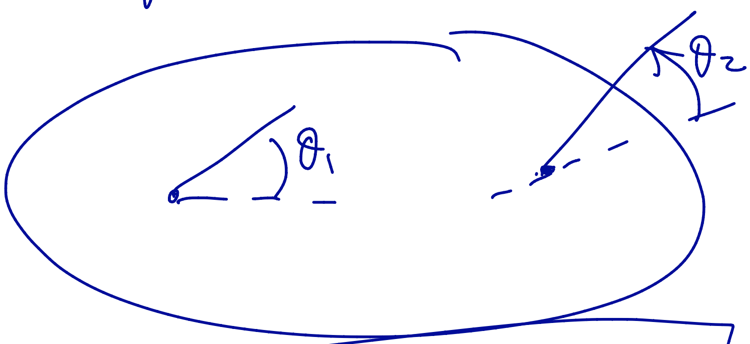
$$\dot{\vec{r}}_{PQ} = \|\vec{r}_{PQ}\|\,\dot{\theta}\,\underbrace{(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})}_{\hat{k}\times(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})}$$

$$\dot{\vec{r}}_{PQ} = (\dot{\theta}\hat{k}) \times \vec{r}_{PQ}$$

We call $\dot{\theta}\hat{k} = \vec{\omega}$ the angular velocity of the rigid body.

$$\Longrightarrow \quad \vec{v}_Q = \vec{v}_P + \vec{\omega} \times \vec{r}_{PQ}$$

Thought experiment



$$\boxed{\theta_1 - \theta_1' = \theta_2 - \theta_2'}$$