Big Picture on Position, Position vectors

Positions (points in space) → D coordinates

 posição vectors

\( \mathbf{r} = \mathbf{v} \times t + \mathbf{a} \times t \)

Cartesian

\[ \mathbf{r} = x(t) \mathbf{i} + y(t) \mathbf{j} \]

\[ \dot{\mathbf{r}} = \dot{x}(t) \mathbf{i} + \dot{y}(t) \mathbf{j} \]

\[ \ddot{\mathbf{r}} = \ddot{x}(t) \mathbf{i} + \ddot{y}(t) \mathbf{j} \]

\[ \dddot{\mathbf{r}} = \dddot{x}(t) \mathbf{i} + \dddot{y}(t) \mathbf{j} \]

Can traverse this chart any way we like.

\[ r = \sqrt{x^2 + y^2} \]

\[ \theta = \tan^{-1}(x, y) \]

Polar

\[ \mathbf{r} = r(t) \hat{r}(t) \]

\[ \dot{\mathbf{r}} = \dot{r}(t) \hat{r}(t) + r(t) \dot{\theta}(t) \hat{\theta}(t) \]

\[ \ddot{\mathbf{r}} = \ddot{r}(t) \hat{r}(t) + (\ddot{\theta}(t) + 2 \dot{\theta}(t)) \hat{\theta}(t) \]

Velocity is the time derivative of the position vector, not the coordinates.

\[ \mathbf{v} = \dot{\mathbf{r}} \]

\[ \mathbf{a} = \ddot{\mathbf{r}} \]

\[ \mathbf{F} = \frac{\partial}{\partial \mathbf{r}} \mathbf{V}(\mathbf{r}) \]

\[ \mathbf{E} = \frac{\partial}{\partial \mathbf{r}} \mathbf{B}(\mathbf{r}) \]
Today our focus is parametrized curves.

**Example:**

\[ y(x) = 50 \cos \left( \frac{2\pi x}{3000} \right) \]

\[ f(x) = 50 \left( \frac{2\pi}{3000}x \right) \sin \left( \frac{2\pi x}{3000} \right) \]

If the driver of the car maintains a constant speed of 55 mph, what is the car's velocity at \( x = 2500 \) ft?

The position vector \( \mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j} \) is given by

\[ \mathbf{v}(t) = \mathbf{r}'(t) = x'(t)\hat{i} + f'(x(t))x'(t)\hat{j} \]

The speed being constant means \( |\mathbf{v}(t)| = 55 \) mph.

Using the relationship between \( x \) & \( y \):

\[ x'(t)^2 + \left( f'(x(t))x'(t) \right)^2 = \left( \frac{55 \text{ mph}}{\text{ft/sec}} \right)^2 \]

\[ x'(t)^2 + \left( \frac{f'(x(t))x'(t)}{x'(t)} \right)^2 = \left( 55 \times \frac{88}{60} \right)^2 (\text{ft/sec})^2 \]

Let \( t_0 \) be the time at which \( x = 2500 \)

\[ x'(t_0) = \frac{(80.7)^2}{1 + f'(2500)^2} = \frac{(80.7)^2}{1 + 55 \times \left( \frac{2\pi}{3000} \right)^2 \sin \left( \frac{2\pi x}{3000} \right)^2} \]

\[ x'(t_0) = 80.3 \text{ ft/sec} \]

\[ \mathbf{v}(t_0) = 80.3 \hat{i} + 7.29 \hat{j} \text{ ft/sec} \]
The Parametric Curve and Decompositions

\( s = \text{distance travelled along the curve.} \)

\( \mathbf{P}(0) \) means the point we are at after travelling \( 0 \) units of distance along the curve.

\( \mathbf{P}(10) \) means the point we are at after travelling \( 10 \) units of distance along the curve.

Example: a straight line

\[ \mathbf{d}(s) = \mathbf{b} + s \cdot \mathbf{a} \]

\[ \left\| \mathbf{d}(s_2) - \mathbf{d}(s_1) \right\| = s_2 - s_1 \]

Unit tangent vector

For small \( s \) we have

\[ (s^2 + \epsilon)^2 \approx \| \mathbf{r}'(s) \mathbf{a} \|^2 \]

\[ \| \mathbf{r}'(s) \|^2 = 1 \] so \( \mathbf{r}'(s) \) is a unit vector.

\[ \hat{\mathbf{e}}_T(s) = \mathbf{r}'(s) \] is a unit tangent vector.

Curvature and the Principal Normal Vector

\[ \frac{d}{ds} \{ \hat{\mathbf{e}}_T(s) \cdot \hat{\mathbf{e}}_T(s) \} = 0 \implies \hat{\mathbf{e}}_T(s) \cdot \mathbf{r}'(s) = 0 \]

\( \hat{\mathbf{e}}_T(s) \) is orthogonal.

\( \| \hat{\mathbf{e}}_T(s) \| \) large means lots of change locally.