Free Body Diagram:

\[ \hat{e}_0' \text{ points into the plane of the page} \]

Newton-Euler equations:

\[
\vec{F}_T + \vec{F}_G = M \ddot{\vec{r}}_{CM/O} \\
-\vec{r}_{CM/P} \times \vec{F}_T = \ddot{\vec{L}}_{CM}
\]
Let \( P \) have spherical polar coordinates \((\ell, \theta, \phi)\) in \( S \).

The associated spherical polar basis vectors at \( P \) are

\[
\hat{e}_r = \cos \theta \cos \phi \hat{i} + \sin \theta \cos \phi \hat{j} + \sin \phi \hat{k}
\]
\[
\hat{e}_\theta = -\sin \theta \hat{i} + \cos \theta \hat{j}
\]
\[
\hat{e}_\phi = -\cos \theta \sin \phi \hat{i} - \sin \theta \sin \phi \hat{j} + \cos \phi \hat{k}
\]

Note that we are measuring \( \phi \) upward from the \( xy \)-plane, not downward from the \( z \)-axis. Let \( CM \) have spherical polar coordinates \((d, \theta', \phi')\) in \( S' \).

The associated spherical polar basis vectors at \( CM \) are

\[
\hat{e}_r' = \cos \theta' \cos \phi' \hat{i} + \sin \theta' \cos \phi' \hat{j} + \sin \phi' \hat{k}
\]
\[
\hat{e}_\theta' = -\sin \theta' \hat{i} + \cos \theta' \hat{j}
\]
\[
\hat{e}_\phi' = -\cos \theta' \sin \phi' \hat{i} - \sin \theta' \sin \phi' \hat{j} + \cos \phi' \hat{k}
\]

We therefore have that

\[
\vec{r}_{cm/0} = \vec{r}_{pl/0} + \vec{r}_{cm/p} = l \hat{e}_r + d \hat{e}_r'
\]
\[
\vec{F}_T = -F_T \hat{e}_r
\]
\[
\vec{F}_o = -Mg \hat{k}
\]
\[
\vec{r}_{cm/p} = d \hat{e}_r'
\]
Now

\[ \mathbf{\Omega}^{CM} = \mathbf{\Omega}^{CM} \cdot \mathbf{\Omega} \]

where \( \mathbf{\Omega}^{CM} \) is the moment of inertia tensor about the center of mass, and \( \mathbf{\Omega} \) is the angular velocity of the gyroscope. The angular velocity has three different components: (1) that due to the spin of the wheel around the axle, (2) that due to changes in \( \phi' \), and (3) that due to changes in \( \theta' \). We may therefore write

\[ \mathbf{\Omega} = \Omega \hat{e}_r' - \phi' \hat{e}_\theta' + \theta' \hat{k} \]

Note that \( \hat{k} \) can be written as \( \hat{k} = \sin \phi' \hat{e}_r + \cos \phi' \hat{e}_\theta \), so that

\[ \mathbf{\Omega} = (\Omega + \theta' \sin \phi') \hat{e}_r' - \phi' \hat{e}_\theta' + \theta' \cos \phi' \hat{e}_\phi' \]
Furthermore, in the \( \{\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi\} \) basis, \( \bar{\mathbf{I}}^{CM} \) is diagonal:

\[
\begin{bmatrix}
I_r' & 0 & 0 \\
0 & I_{\theta\theta}' & 0 \\
0 & 0 & I_{\phi\phi}'
\end{bmatrix}
\]

where \( I_r' \) is the principal moment of inertia about the axle, and \( I_{\theta\theta}' = I_{\phi\phi}' \) are the other two principal moments of inertia. For simplicity, write \( I_r' = I_r \), \( I_{\theta\theta}' = I_{\phi\phi}' = I_0 \). Then

\[
\begin{bmatrix}
I & 0 & 0 \\
0 & I_0 & 0 \\
0 & 0 & I_0
\end{bmatrix}
\begin{bmatrix}
\Omega + \dot{\phi} \sin \phi \\
-\dot{\phi} \\
\dot{\phi} \cos \phi
\end{bmatrix}
\]

Hence, we have that

\[
\bar{\mathbf{c}}^{CM} = \mathbf{I} \left( \Omega + \dot{\phi} \sin \phi \right) \hat{e}_r' - I_0 \dot{\phi} \hat{e}_\theta' + I_0 \dot{\phi} \cos \phi \hat{e}_\phi'
\]

(b) Six scalar equations

(c) Six unknowns: \( \theta, \phi, \theta', \phi', \Omega, F_r \)

(d) Yes; eliminate \( F_r \) to get a system of 5 ordinary differential equations in \( \theta, \phi, \theta', \phi', \Omega \), and solve numerically.