TAM 212 Worksheet 7: Car steering

This worksheet aims to understand how cars steer. The #avs webpage on “Steering geometry” illustrates the basic ideas. On the diagram below, the kingpins at A and B are a distance \( g \) apart (this is almost the same as the track distance between the front wheel tire centers), while the wheelbase distance is \( AF = BE = \ell \). Modern cars use ball joints instead of actual pins at the kingpin joints.

1. Consider the four-wheeled car configuration shown above. The left-front wheel is turned at an angle of \( \theta_L \), and the turning radius of the car is \( \rho \), measured from the center \( P \) of the rear axle to the instantaneous center \( M \). Derive a formula for \( \rho \) in terms of \( \theta_L \), leaving measurements \( g \) and \( \ell \) in symbolic form.

2. Similarly to the previous question, derive a formula for \( \rho \) in terms of the angle \( \theta_R \) of the right-front wheel.
3. While trying to park our car in a tight spot, we want to drive our car around a counter-clockwise curve with a radius of curvature of $\rho = 5$ m. At what angles $\theta_L$ and $\theta_R$ should we ideally set our wheels, in order to make this turn? Give your answers in numeric form.

4. Ackermann steering geometry, shown in the figure below, uses a four-bar linkage $ABCD$ to constrain the wheel angles $\theta_L$ and $\theta_R$. The tie rod has length $CD = f$, while the steering arms have lengths $AD = a$ and $BC = b$. A simple rule of thumb for designing Ackermann steering sets the linkage geometry so that the steering arms point to the center $P$ of the rear axle, as shown. Given lengths $a = b = 0.2$ m, what is the angle $\gamma$ and the appropriate length $f$ of the tie rod?

\[ g = 2 \text{ m} \]
\[ \ell = 3 \text{ m} \]
5. The initial and turned state of front wheels of Ackermann steering geometry case are drawn as below. On this figure, indicate the turning angles of right and left wheel ($\theta^*_R$ and $\theta^*_L$), and $\gamma$.

6. $\theta_L$ and $\theta_R$ we found from Q3 are ideal angles during the turn without the tie rod $CD$. We want to see how well the Ackermann steering geometry we designed in the Q4 works. Consider the turn from Q3 with $\rho = 5$ m, and set the left-front wheel angle $\theta^*_L$ is equal to the value $\theta_L$ found in Q3 ($\theta^*_L = \theta_L$).

What right-front angle $\theta^*_R$ is now determined by the linkage? Use the diagram below to start with $\theta_L$ and work your way across the diagram to find $\theta^*_R$. The law of cosines will be helpful for determine angles on general triangles, for example

$$c^2 = a^2 + g^2 - 2ag \cos \angle DAB$$
7. How close is the Ackermann value of $\theta_R^{*}$ from Q5 to the ideal value $\theta_R$ from Q3? Is this Ackermann steering geometry acceptable for real-world usage?

8. Bonus question: While making the turn in the above question, we measure the speed of point $P$ to be $v_P = 2$ m/s (we are parking very quickly!). What is the angular velocity $\omega$ of the car during the turn?

9. Bonus question: While turning as above, what are the speeds of the four wheel joints $v_A$, $v_B$, $v_E$, and $v_F$?

10. Bonus question: Considering the velocities of the four wheel joints you found in Q8, would it be reasonable to build a rear-wheel-drive car with a rear driveshaft consisting of a single rod bolted to each wheel? Why or why not? What might an alternative be?