TAM 212 Worksheet 8: Accelerating and braking

The aim of this worksheet is to understand how vehicles accelerate and brake, and how traction and contact limit the maximum acceleration in both cases.

The photo above shows a 1965 Ford Mustang Fastback, which uses real-wheel drive. This means that the engine powers the back wheels of the car, while the front wheels are used for steering. Braking is done by all wheels. The dimensions of the car are shown below, where $C$ is the center of mass.
Equations of motion for the car model

We will use a multiple rigid body model of the car to analyze the car, as shown in the side-view free-body diagram below. The car contacts the ground at two points (rear and front wheels) with normal forces $N_1$ and $N_2$ and the friction force $F_1$ acts only on the rear (driving) wheels. Gravity $g$ acts downwards through the center of mass $C$. Although there are actually four wheels, we consider the rear wheels together, so $F_1$ and $N_1$ are the total forces acting on the rear wheels, and $M_1$ is the total moment. Note that $M_1$ is a pure moment.

![Free-body diagram of a car](image)

1. We will assume that the wheels have negligible mass and negligible moment of inertia, so $\sum F = 0$ and $\sum M = 0$ for each wheel (moments taken about the center of each wheel). Using this, determine $N_1, N_2, F_1, M_1$ in terms of $R_1, R_2, D_1$. 
2. Assume the car is accelerating while remaining in contact with the road, so the car body is not rotating. Compute the forces $R_1, R_2, D_1$ and the moment $M_1$ as functions of $a$ and the dimensions of the car.

3. Combine your answers from Questions 1 and 2 to find the forces $F_1, N_1, N_2$ in terms of $a$ and the dimensions $m, \ell, H$ of the car. You should eliminate $M_1, D_1, R_1, R_2$. 
Accelerating and staying on the ground

Below is an image of a 1995 Ford Mustang Cobra that is accelerating so hard that its front wheels have lost contact with the ground (the car is performing a wheelstand or wheelie). Apart from any other problems, this means that the car is completely unable to steer.

4. What is the maximum acceleration $a$ possible before the front wheels lose contact with the ground? Express your answer in terms of $g$ and the dimensions $\ell, H$ of the car. What is this maximum acceleration for a Mustang with $\ell = 1.41$ m and $H = 0.60$ m?

5. If you want to design a car to minimize the risk of losing ground contact while accelerating, should $\ell$ and $H$ be large or small? Does this make sense physically?
Maintaining traction while accelerating

The photo below shows a Ford Mustang GT that has lost traction while accelerating (it is experiencing a burnout). While it was probably intentional in this case, loss of traction is major problem in practice, and it can severely limit the maximum acceleration.

6. The friction force $F_1$ on the rear wheels is limited by $F_1 \leq \mu N_1$, where $\mu$ is the coefficient of friction. What is the maximum acceleration $a$ before the rear wheels start to slip? Assume the front wheels remain in contact with the ground when computing your answer, and express it in terms of $\mu, g, \ell, H$.

7. Taking $\mu = 0.7$ and the dimensions $\ell = 1.41$ m and $H = 0.60$ m, is the maximum acceleration of our car model limited by maintaining front-wheel ground contact or maintaining real-wheel traction? What is the maximum acceleration?
8. If we can change the tires on our car to increase the friction coefficient \( \mu \), what value of \( \mu \) should we choose to have the maximum possible acceleration? What is this maximum acceleration?

9. From your answer to Question 8, is it physically plausible that a car could flip while accelerating? Explain.  
   Hint: Dragster tires can achieve friction coefficients of up to \( \mu \approx 4 \).