Announcements

- Students are encouraged to practice drawing FBDs, writing out equilibrium equations, and solving these by hand using your calculator.
- Expending large amounts of time trying to de-bug MATLAB code, especially during a quiz, is not the focus of this course. All problems can be solved by hand. Quiz questions are timed for solution by hand.

Upcoming deadlines:
- Friday (10/19)
  - Written Assignment 4
- Tuesday (10/23)
  - Prairie Learn HW5
- Quiz 2 (10/24) **Wednesday**
  - During class time (9:00 am)
  - Computer Lab (D211 for ME, D331 for CEE)
- No class
  - Friday October 26 (Sports Meeting day)
  - Monday October 29

PrairieLearn incorrect software issues:
- Negative sign symbol (- vs. – )
- Space between negative sign (-12 vs. - 12)

Solutions:
- Always type in the negative sign symbol (-) into your PL answers for HW or Quiz.
- Do not add space between negative symbol and number
- All students with these errors will be provided updated grades on Quiz 1. No credit for Quiz 2 and beyond.
Chapter 5: Equilibrium of Rigid Bodies
Goals and Objectives

- Introduce the free-body diagram for a rigid body
- Develop the equations of equilibrium for a 2D and 3D rigid body
- Solve rigid body equilibrium problems using the equations of equilibrium in 2D and 3D

- Introduce concepts of
  - Support reactions for 2D and 3D bodies
  - Two- and three-force members
  - Constraints and statical determinacy
Recap: \textbf{Equilibrium of a Rigid Body}

Reduce forces and couple moments acting on a body to an equivalent resultant force and a resultant couple moment at an arbitrary point \( O \).

\[
\begin{align*}
\sum F_x &= 0 \quad \sum F_y &= 0 \\
\sum M_O &= 0 \\
\sum F &= 0 \quad \sum M &= 0
\end{align*}
\]
Process of solving rigid body equilibrium problems

1. Create idealized model (model and assumptions)

2. Draw free body diagram showing ALL the external (applied loads and support reactions)

3. Apply equations of equilibrium

\[ \sum \vec{F} = 0 \]
\[ \sum \vec{M}_A = 0 \]

Let’s sum moments about pt A. **Pick pt to sum moments that eliminates as many unknowns as possible.**

[See Example 5.11 in text for full derivation]

Assume uniform truck ramp of 400 lb held by 2 cables

This slide presents the basic approach for problem solving for this course (previous slide). Understand how to do this approach!
Recap: Equilibrium in two-dimensional bodies

(Support reactions)

- **Roller**
- **Smooth pin or hinge**
- **Fixed support**

- If a support prevents the translation of a body in a given direction, then a force is developed on the body on that direction.
- If a rotation is prevented, a couple moment is exerted on the body.
Beam has mass of 100 kg and experiences load of 1200 N.
Identify support reaction type.
Find support reactions at A.

Given: \( m = 100 \text{ kg}, F = -1200 \text{ N} \)

Find: \( A_x, A_y, M_A \)

1. FBD for beam:

2. Write E.o.E

\[ \Sigma F_x = 0: \quad A_x = 0 \]

\[ \Sigma F_y = 0: \quad A_y - F - W = 0, \quad A_y = 1200 \text{ N} - 100 \text{ kg} (9.81 \text{ m/s}^2) = 0 \]

\[ A_y = 2180 \text{ N} \]

\[ \Sigma M_A = 0: \quad M_A - (2 \text{ m}) F - (\frac{5}{2} \text{ m}) W = 0, \quad M_A - (2 \text{ m})1200 \text{ N} - (5 \text{ m})981 \text{ N} = 0 \]

\[ M_A = 5340 \text{ N m} \text{ ccw} \]

See Example 5.1 in text
The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb. Draw the FBD of the pedal.

\[ \Sigma F_x = 0 : \]
\[ \Sigma F_y = 0 : \]
\[ \Sigma M_A = 0 : \]

Solve for \( A_x, A_y, F \).

Assume pedal is massless

No weight

\[ F_s = k \Delta y \]
\[ = (20 \text{ lb})(1.5 \text{ in}) \]
\[ = 30 \text{ lb} \]

See Example 5.2 in text.
Identify support reaction types. Draw the FBD of body AB with forces in Cartesian coordinates.

Uniform beam of 200 lb

Massless truss

No weight since massless

Pinned

Roller
Massless beam

25 kg bar with center of mass at G. Supported by smooth peg at C, roller at A, cord AB

Identify support reaction types. Draw the FBD of rigid body with forces in Cartesian coordinates.
Two-force members

As the name implies, two-force members have forces applied at only two points.

If we apply the equations of equilibrium to such members, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.
Examples of two-force members

In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

Two-force members simplify the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).
As the name implies, three-force members have forces applied at only three points.

Moment equilibrium can be satisfied only if the three forces are concurrent or parallel force system.

Three-force member: a force system where the three forces (a) meet at the same point (point O), or (b) are parallel.
Two-force and three-force members

One can use these concepts to quickly identify the direction of an unknown force.

Two-force member: the two forces at ends are equal, opposite, collinear

Three-force member: a force system where the three forces
1. meet at the same point (point O), or
2. are parallel

See Example 5.13 in text
Draw FBDs for each two or three force member (BC, JK, IE, IG, Bucket). Ignore weight of each link. Include dirt weight in bucket.

Line of action of an unknown force can be determined from 2- or 3-force members:

2-force member:
The 2 forces at ends are equal, opposite, collinear.

3-force member: force system where the 3 forces meet at the same point, or are parallel.

Directions of arrows of unknown forces/moments are arbitrary on FBD. Actual direction will be determined after solving for unknown values.
Constraints

To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports.

- **Redundant constraints**: the body has more supports than necessary to hold it in equilibrium; the problem is **STATICALLY INDETERMINATE** and cannot be solved with statics alone. Too many unknowns, not enough equations.
Constraints

- **Improper constraints:** In some cases, there may be as many unknown reactions as there are equations of equilibrium (statically determinate). However, if the supports are not properly constrained, the body may become unstable for some loading cases.
  - BAD: Reactive forces are concurrent at same point (point A) or line of action
  - BAD: Reactive forces are parallel

Stable body: lines of action of reactive forces do not intersect at common axis, and are not parallel