Announcements

- Upcoming deadlines:
  - Tuesday (Sept 18)
    - HW1
    - Find on PrairieLearn
  - Friday (Sept 21)
    - Written Assignment 1
    - Find on Schedule
    - Submit on Blackboard
Recap from Lecture 3

- **Position vector**

\[
\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B \mathbf{i} + y_B \mathbf{j} + z_B \mathbf{k}) - (x_A \mathbf{i} + y_A \mathbf{j} + z_A \mathbf{k})
\]

\[
\mathbf{r} = (x_B - x_A) \mathbf{i} + (y_B - y_A) \mathbf{j} + (z_B - z_A) \mathbf{k}
\]

Thus, the \((i, j, k)\) components of the position vector \(\mathbf{r}\) may be formed by taking the coordinates of the tail (point A) and subtracting them from the corresponding coordinates of the head (point B).
The force vector $\mathbf{F}$ acting along the rope can be defined by the unit vector $\mathbf{u}$ (defined the direction of the rope) and the magnitude $F$ of the force.

The unit vector $\mathbf{u}$ is specified by the position vector $\mathbf{r}$:
Force vector directed along a line

Determine the force vector $\vec{F}$ along the rope.
Dot (or scalar) product

The dot product of vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as such

$$\mathbf{A} \cdot \mathbf{B} =$$

Laws of operation:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$
$$\alpha(\mathbf{A} \cdot \mathbf{B}) = \alpha \mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \alpha \mathbf{B}$$
$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$$

Cartesian vector formulation:

Note that:

$$\mathbf{i} \cdot \mathbf{j} = 0 \quad \mathbf{i} \cdot \mathbf{i} = 1$$
Projection of vector onto parallel and perpendicular lines

The scalar component \( A_\parallel \) of a vector \( \mathbf{A} \) along (parallel to) a line with unit vector \( \mathbf{u} \) is given by:

\[
A_\parallel = \mathbf{A} \cdot \mathbf{u}
\]

And thus the vector components \( \mathbf{A}_\parallel \) and \( \mathbf{A}_\perp \) are given by:
Determine the projected component of the force vector $F_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector.
The cross product of vectors \( \mathbf{A} \) and \( \mathbf{B} \) yields the vector \( \mathbf{C} \), which is written

\[
\mathbf{C} = \mathbf{A} \times \mathbf{B}
\]

The magnitude of vector \( \mathbf{C} \) is given by:

\[
|\mathbf{C}| = |\mathbf{A}| |\mathbf{B}| \sin \theta
\]

The vector \( \mathbf{C} \) is perpendicular to the plane containing \( \mathbf{A} \) and \( \mathbf{B} \) (specified by the right-hand rule). Hence,

Geometric definition of the cross product: the magnitude of the cross product is given by the area of a parallelogram

\[
|\mathbf{C}| = |\mathbf{A}| |\mathbf{B}| \sin \theta
\]
Laws of operation:

\[ A \times B = -B \times A \]

\[ \alpha (A \times B) = (\alpha A) \times B = A \times (\alpha B) = (A \times B) \alpha \]

\[ A \times (B + D) = A \times B + A \times D \]
The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$.

Considering the cross product in Cartesian coordinates

$A \times B$
Cross (or vector) product

Also, the cross product can be written as a determinant.

\[
A \times B = \begin{vmatrix}
i & j & k \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

Each component can be determined using \(2 \times 2\) determinants.

For element \(i\):

\[
\begin{vmatrix}
i & j & k \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

For element \(j\):

\[
\begin{vmatrix}
i & j & k \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

For element \(k\):

\[
\begin{vmatrix}
i & j & k \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

\[
A \times B = (A_y B_z - A_z B_y)i - (A_x B_z - A_z B_x)j + (A_x B_y - A_y B_x)k
\]