Announcements

- Take practice Quiz 0 on PrairieLearn (not graded)
- MATLAB training sessions TBA (Friday afternoon next 2 weeks)

- Upcoming deadlines:
  - Tuesday (Sept 18)
    - HW1
    - Find on PrairieLearn
  - Friday (Sept 21)
    - Written Assignment 1
    - Find on Schedule
    - Submit on Blackboard
Chapter 2: Force vectors
Main goals and learning objectives

Define scalars, vectors and vector operations and use them to analyze forces acting on objects

- Add forces and resolve them into components
- Express force and position in Cartesian vector form
- Determine a vector’s magnitude and direction
- Introduce the dot product and use it to find the angle between two vectors or the projection of one vector onto another
Recap from Lecture 2

- A force can be treated as a vector, since forces obey all the rules that vectors do.

\[ \mathbf{R} = \mathbf{A} + \mathbf{B} \]
\[ \mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \]
\[ \mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \]
Recap

- **Vector representations**
  - **Rectangular components**
  - **Cartesian vectors**
  - **Unit vector**

Recall: Magnitude of a vector (which is a scalar quantity) can be shown as a term with no font modification ($A$) or vector with norm bars ($\vec{A}$), such that $A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
\[ \vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z \quad \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \]

- How to define \( A_x, A_y, A_z \)?

- Direction cosines
  \[ \cos(\alpha) = \frac{A_x}{A}, \cos(\beta) = \frac{A_y}{A}, \cos(\gamma) = \frac{A_z}{A} \]
  \[ \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \]
  \[ = A \cos(\alpha) \hat{i} + A \cos(\beta) \hat{j} + A \cos(\gamma) \hat{k} \]

\( \alpha, \beta, \gamma \) are angles in 3D

- Rectangular components
  \[ A_x = A \cos(\theta), \quad A_y = A \sin(\theta) \]
  \[ A_x = A \left( \frac{a}{c} \right), \quad A_y = A \left( \frac{b}{c} \right) \]
The cables attached to the screw eye are subjected to three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = A \left[ \cos(\alpha) \hat{i} + \cos(\beta) \hat{j} + \cos(\gamma) \hat{k} \right] \]

(b) Determine the magnitude of the resultant force vector

\[ |\vec{F}_R| = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2} \]

(c) Determine the direction cosines of the resultant force vector

\[ \cos(\alpha) = \frac{A_x}{A}, \quad \cos(\beta) = \frac{A_y}{A}, \quad \cos(\gamma) = \frac{A_z}{A} \]
The cables attached to the screw eye are subjected to three forces shown.
(a) Express each force vector using the Cartesian vector form (components form).

\[ \mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} = A [\cos(\alpha) \mathbf{i} + \cos(\beta) \mathbf{j} + \cos(\gamma) \mathbf{k}] \]

(b) Determine the magnitude of the resultant force vector

\[ |\mathbf{F}_R| = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2} \]

\[ F_1 = (350 \text{ N}) [\cos(90^\circ) \mathbf{i} + \cos(50^\circ) \mathbf{j} + \cos(40^\circ) \mathbf{k}] \]
\[ F_2 = (150 \text{ N}) [\cos(45^\circ) \mathbf{i} + \cos(60^\circ) \mathbf{j} + \cos(120^\circ) \mathbf{k}] \]
\[ F_3 = (250 \text{ N}) [\cos(60^\circ) \mathbf{i} + \cos(135^\circ) \mathbf{j} + \cos(60^\circ) \mathbf{k}] \]

\[ \mathbf{F}_1 = (225 \mathbf{j} + 264 \mathbf{k}) \text{ N} \]
\[ \mathbf{F}_2 = (70.7 \mathbf{i} + 50.0 \mathbf{j} - 50.0 \mathbf{k}) \text{ N} \]
\[ \mathbf{F}_3 = (125 \mathbf{i} - 177 \mathbf{j} + 125 \mathbf{k}) \text{ N} \]

\[ \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = F_{Rx} \mathbf{i} + F_{Ry} \mathbf{j} + F_{Rz} \mathbf{k} \]
\[ = (0 + 70.7 + 125) \mathbf{i} + (225 + 50 - 177) \mathbf{j} + (269 - 50 + 125) \mathbf{k} \text{ N} \]
\[ = 195.71 \mathbf{i} + 98.20 \mathbf{j} + 343.12 \mathbf{k} \text{ N} = |\mathbf{F}_x| \mathbf{i} + |\mathbf{F}_y| \mathbf{j} + |\mathbf{F}_z| \mathbf{k} \]
\[ |\mathbf{F}_x| = \sqrt{(195.71)^2 + (98.20)^2 + (343.12)^2} = 407.03 \text{ N} \]
\[ |\mathbf{F}_y| = 407 \text{ N} \]
\[ |\mathbf{F}_z| = 407 \text{ N} \]
The cables attached to the screw eye are subjected to three forces shown.
(c) Determine the direction cosines of the resultant force vector

\[
\cos(\alpha) = \frac{A_x}{A}, \quad \cos(\beta) = \frac{A_y}{A}, \quad \cos(\gamma) = \frac{A_z}{A}
\]

\[
\vec{F}_R = \vec{F}_{Rx} + \vec{F}_{ Ry} + \vec{F}_{ Rz}
\]

\[
\cos(\alpha_R) = \frac{|\vec{F}_{Rx}|}{|\vec{F}_R|} = \frac{195.71}{407.03}
\]

\[
\cos(\beta_R) = \frac{F_{Ry}}{F_R} = \frac{98.20}{407.03}
\]

\[
\cos(\gamma_R) = \frac{F_{Rz}}{F_R} = \frac{343.12}{407.03}
\]
A position vector \( \mathbf{r} \) is defined as a fixed vector which locates a point in space relative to another point. For example,

\[
\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}
\]

expresses the position of point \( P(x, y, z) \) with respect to the origin \( O \).

The position vector \( \mathbf{r} \) of point \( B \) with respect to point \( A \) is obtained from:

\[
\mathbf{r}_B = \mathbf{r}_A + \mathbf{r} = \mathbf{r}_B - \mathbf{r}_A
\]

Thus, the \((i, j, k)\) components of the position vector \( \mathbf{r} \) may be formed by taking the coordinates of the tail (point \( A \)) and subtracting them from the corresponding coordinates of the head (point \( B \)).
Example

Determine the lengths of bars $AB$, $BC$ and $AC$.

\[ \vec{r}_{AB} = \vec{r}_B - \vec{r}_A \]

\[ \vec{r}_A = 0.8\hat{i} + 1.2\hat{j} \text{ [m]} \]

\[ \vec{r}_B = ? \]

\[ = (0.8 + 0.3 + q)\hat{i} + 1.5\hat{j} \text{ [m]} \]

$q =$ ?

Use Right Triangle:

\[ \tan 40^\circ = \frac{opp}{adj} = \frac{1.5}{q} \]

\[ q = 1.5 / \tan 40^\circ = 1.79 = 1.8 \text{ m} \]

\[ \vec{r}_B = 2.9\hat{i} + 1.5\hat{j} \]

\[ \vec{r}_{AB} = 2.1\hat{i} + 0.3\hat{j} \]

\[ AB = |\vec{r}_{AB}| = \sqrt{(2.1)^2 + (0.3)^2} \quad = 2.1 \text{ m} \]

\[ AB = 2.1 \]

Solve for $BC$ & $AC$ on your own.