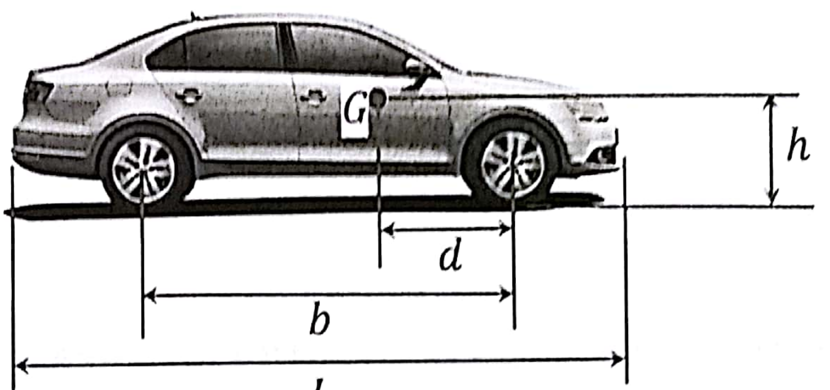
Name: ____________

Group members: ____________ ____________ ____________

TAM 210/211 - Worksheet 11

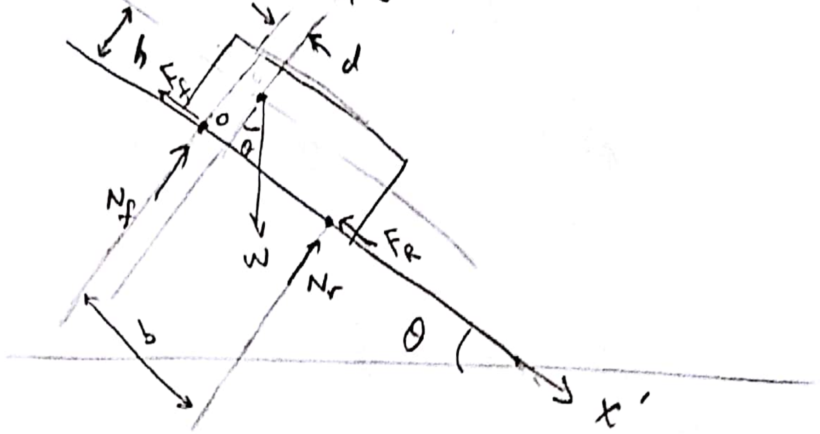
Today we will analyze how vehicles perform on slopes in a variety of conditions. We will consider upward and downward slopes and vehicles that are braked or have slipping wheels. We want to consider design options to improve performance, so as you work the problems, solve everything analytically, and only use numbers at the very end.

A vehicle makes contact with the road at both the front and rear tires; those tires experience static friction when not slipping, and kinetic friction if the wheels are slipping. We'll consider the 2013 Volkswagen Jetta TDI: wheelbase $b = 104.4$ in, center of gravity defined by $h = 22.5$ in and $d = 40.5$ in, total weight $W = 3255$ lb and front-wheel drivetrain.



We want to consider what the largest slopes – pointing uphill and downhill that this vehicle can handle when placed on (a) dry pavement, and (b) icy pavement. For everything ahead, we will simplify the problem by considering it in 2D: so the front wheels act together and the rear wheels act together.

1. Draw the free-body diagram for the vehicle uphill on an incline of angle θ. Include and label all the relevant forces.



2. Write equations of equilibrium for this 2D problem, **use symbols only**. Consider using a coordinate system aligned with the inclined plane. For example, x' parallel to the inclined plane and y' perpendicular to the inclined plane, and perform $\sum F_{x'} = 0$ and $\sum F_{y'} = 0$.

(1) $\sum F_{x'} = 0 \Rightarrow F_f + F_r = W \sin \theta$

(2) $\sum F_{y'} = 0 \Rightarrow N_f + N_r = W \cos \theta$

(3) $\sum M_o = 0 \Rightarrow (N_r) b - (W \cos \theta) d - (W \sin \theta) h = 0$

3. Obtain symbolic expressions for the normal forces on the front and rear tires as a function of θ and the given variables.

From (3) : $N_r = \frac{W}{b} (d \cos \theta + h \sin \theta)$ (4)

sub N_r into (2) : $N_f = W \cos \theta - \frac{W}{b} (d \cos \theta + h \sin \theta)$ (5)

4. Assume that brakes are engaged: all tires experience static friction. Obtain an expression for the maximum angle for equilibrium.

$*$ static friction + impending motion. $\Rightarrow$ $F_f = \mu_s N_f$
$F_r = \mu_s N_r$

From ① : $\mu_s N_f + \mu_s N_r = W \sin\theta$

using ④ and ⑤ $\Rightarrow$ $\mu_s (W\cos\theta) = W\sin\theta.$

$\mu_s = \tan\theta$

$\theta = \tan^{-1}(\mu_s)$ #

5. What is the value of the maximum angle for equilibrium if the Jetta is on dry pavement ($\mu_s = 0.75$)? Or in icy pavement ($\mu_s = 0.24$)?

$\mu_s = 0.75 \Rightarrow \theta = 36.9°$

$\mu_s = 0.24 \Rightarrow \theta = 13.5°$. #

6. Next, consider only the hand brake is engaged, so the front wheels experience static friction, but the rear tires can freely rotate (i.e. no contribution from static friction). Obtain the symbolic expression for the maximum angle for equilibrium.

front wheel : $F_f = \mu_s N_f$

rear Wheel : $F_r = 0$

Eqn. 1 becomes $F_f = \mu_s N_f = w \sin\theta$. $\Rightarrow$ $N_f = \dfrac{W\sin\theta}{\mu_s}$

From eqn. 5 $N_f = W\cos\theta - \dfrac{W}{b}(d\cos\theta + h\sin\theta) = \dfrac{W\sin\theta}{\mu_s}$.

$$\left(1 - \frac{d}{b}\right)\cos\theta = \left(\frac{1}{\mu_s} + \frac{h}{b}\right)\sin\theta.$$

$$\theta = \tan^{-1}\left[\frac{1 - \frac{d}{b}}{\frac{1}{\mu_s} + \frac{h}{b}}\right] \quad \#$$

7. What is the value of the maximum angle for equilibrium if the Jetta is on dry pavement ($\mu_s = 0.75$)? Or in icy pavement ($\mu_s = 0.24$)?

$\mu_s = 0.75 \Rightarrow \theta = 21.56°$.

$\mu_s = 0.24 \Rightarrow \theta = 7.95°$.

8. What changes would you make to the car to increase the possible incline angle?

- decrease h (center of gravity closer to ground)
- decrease d (center of gravity closer to front wheel)
- increase μ_s (better traction)

9. What is the maximum incline so that the car does not flip backwards?

about to tip : $N_f = 0$

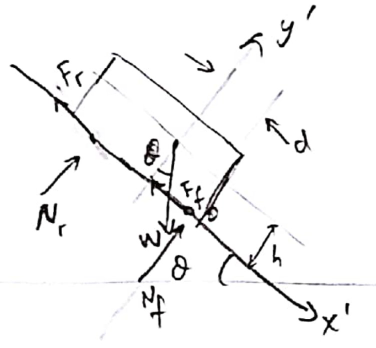
From 2 & 4 $N_r = W\cos\theta$

$$\frac{W}{b}(d\cos\theta + h\sin\theta) = W\cos\theta$$

$$\cos\theta\left(1 - \frac{d}{b}\right) = \frac{h}{b}\sin\theta$$

$$\theta = \tan^{-1}\left[\frac{1 - d/b}{h/b}\right] \Rightarrow \boxed{\theta_{max} = 70.6^\circ}$$

10. *Challenge.* Does your analysis change if the vehicle is pointed downhill instead? Explain.



only eqn. 3 changes

$$-(N_r)b + (W\cos\theta)d - (W\sin\theta)h = 0$$

$$N_r = \frac{W}{b}(d\cos\theta - h\sin\theta)$$

Part 4) does not change since result does not depend on eqn. 3 (independent of the centroid position)

Part 6) From eqn 5, $N_f = \dfrac{W\sin\theta}{\mu_s} = W\cos\theta - \dfrac{W}{b}(d\cos\theta - h\sin\theta)$

different!

$$\theta = \tan^{-1}\left[\frac{1 - d/b}{\frac{1}{\mu_s} - h/b}\right]$$

* Now increasing h will increase maximum angle! (more friction at front wheel!)