TAM 210/211 - Worksheet 2

Objectives:

• Write forces as Cartesian vectors.
• Perform vector calculations, such as summation, dot and cross product.
• Write unit vectors.
• Understand some important applications of vectors in engineering.

1) Mechanical equilibrium is a major focus of statics. Examples of equilibrium in everyday life include sitting in a chair and a stack of books resting on a table. What are examples of things that were designed to be in equilibrium?

Answers may vary.

2) For vectors \( \mathbf{A} = 9\mathbf{i} - 5\mathbf{j} \) and \( \mathbf{B} = -2\mathbf{i} + 4\mathbf{j} \), determine:
   a) an expression for the resultant vector \( \mathbf{C} = \mathbf{A} + \mathbf{B} \).
   b) the magnitude and direction angle of the resultant vector \( \mathbf{C} \).
   c) Make a graphical representation of your results.

   a) \( \mathbf{C} = 7\mathbf{i} - \mathbf{j} \)

   b) \( \mathbf{C} = 5\sqrt{2} \), \( \theta = \tan^{-1}\left(\frac{1}{7}\right) \)

3) What is the unit vector that points along \( \mathbf{A} = -1\mathbf{i} - 8\mathbf{j} \)?

   \( \mathbf{\hat{u}}_A = \frac{-1}{\sqrt{65}}\mathbf{i} - \frac{8}{\sqrt{65}}\mathbf{j} \)
4) Two forces act on the hook as indicated below. Assume that the resultant force, $\mathbf{F}_R$, acts along the positive $z$-axis and has magnitude of 600 N. Note that $\mathbf{F}_2$, is a general vector and not meant to indicate any particular direction or magnitude.

a) Express the force $\mathbf{F}_1$ as a Cartesian vector.

\[
\vec{F}_1 = (300 \cos 45^\circ \hat{i} + 300 \cos 60^\circ \hat{j} + 300 \cos 120^\circ \hat{k}) \text{ N}
\]

\[
\vec{F}_1 = (150 \sqrt{2} \hat{i} + 150 \hat{j} - 150 \hat{k}) \text{ N}
\]

b) Express the force $\mathbf{F}_R$ as a Cartesian vector.

\[
\vec{F}_R = 600 \hat{k} \text{ N}
\]

c) Find the magnitude of $\mathbf{F}_2$ and its unit vector.

\[
\vec{F}_2 = \vec{F}_R - \vec{F}_1 = (-150 \sqrt{2} \hat{i} - 150 \hat{j} + 750 \hat{k}) \text{ N}
\]

\[
F_2 = 300 \sqrt{7} \text{ N}
\]

\[
\hat{u}_2 = \frac{-\sqrt{2}}{2 \sqrt{7}} \hat{i} - \frac{1}{2 \sqrt{7}} \hat{j} + \frac{5}{2 \sqrt{7}} \hat{k}
\]
d) Making sure that your designs are able to handle their required loads is an important aspect of engineering. Suppose that the hook only handles the force $F_1$, and that the hook can withstand 100 N along the negative z-axis before failure. Does the hook fail? What are some potential improvements that could be made to the hook’s design?

Yes, the hook will fail ($F_{1z} > 100 \text{N}$).

5) Two forces act on the ring located at point $A$ as indicated below.

a) Express the force $F_1$ as a Cartesian vector.

$$F_1 = 80 \text{ lb } \left( \frac{-25}{4.63} \hat{i} - \frac{4}{7.63} \hat{j} + \frac{6}{7.63} \hat{k} \right)$$
b) Express the force $\mathbf{F}_2$ as a Cartesian vector.

$$\mathbf{F}_2 = 50 \text{ lb} \left( \frac{2}{7.48} \hat{i} - \frac{4}{7.48} \hat{j} - \frac{6}{7.48} \hat{k} \right)$$

c) Write the unit vector that indicates the direction of the resultant force, $\mathbf{F}_R$.

$$\mathbf{F}_R = -0.174\hat{i} - 0.934\hat{j} + 0.310\hat{k}$$

d) Determine the coordinate direction angles for $\mathbf{F}_R$, assuming that the origin is located at A.

$$\alpha = 100.1^\circ$$
$$\beta = 159.1^\circ$$
$$\gamma = 71.9^\circ$$
Looking Ahead

6) For vectors $\mathbf{A} = -5 \mathbf{i} + 1 \mathbf{j} - 8 \mathbf{k}$ and $\mathbf{B} = 6 \mathbf{i} - 3 \mathbf{j} + 4 \mathbf{k}$, what is the cross (vector) product $\mathbf{A} \times \mathbf{B}$?

$$-20 \mathbf{i} - 28 \mathbf{j} + 9 \mathbf{k}$$

7) For vectors $\mathbf{A} = -1 \mathbf{i} + 6 \mathbf{j}$ and $\mathbf{B} = 4 \mathbf{i} - 4 \mathbf{j}$, what is the component of $\mathbf{A}$ onto $\mathbf{B}$?

$$-\frac{7}{\sqrt{2}}$$