Announcements

- Quiz 2 re-try this week (Thu-Sat)
  - Same material, different problems
  - Come to my office hours for Quiz 2 questions
- No lecture Friday (2/15)
  - Friday office hours will still meet

Upcoming deadlines:
- Friday (2/15)
  - Written Assignment
- Tuesday (2/19)
  - PL HW
Objective

- Free body diagram for 2D rigid body
- Types of constraints
- Equations of equilibrium for 2D rigid body
Equilibrium in two-dimensional bodies

Why different support?
Equilibrium in two-dimensional bodies

Active Forces vs. Support reaction components

60°
Free Body Diagrams

1. FBD for frame AB

2. FBD for disk E

3. FBD for AB
Constraints

To ensure equilibrium of a rigid body, it is not only necessary to satisfy equations of equilibrium, but the body must also be properly constrained by its supports.

- **Redundant constraints**: the body has more supports than necessary to hold it in equilibrium; the problem is **STATICALLY INDETERMINATE** and cannot be solved with statics alone.

  **Equilibrium Equation:**
  \[
  \sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0
  \]

- **Improper constraints**: In some cases, there may be as many unknown reactions as there are equations of equilibrium. However, if the supports are not properly constrained, the body may become unstable for some loading cases.

\[
\sum M_A = \vec{r}_P \times \vec{F} \neq 0
\]

- **equilibrium cannot be achieve**
Constraints

Proper, redundant, or improper constraints

\( \sum F_x = 0 = P - A_x = 0 \checkmark \)
\( \sum M_A = \bar{r}_p \times \bar{F} + \bar{r}_B \times \bar{B} + \bar{r}_C \times \bar{C} = 0 \checkmark \)
\( (-\ell) \quad (+\ell) \quad (+\ell) \)
\( \sum F_y = B + C - A = 0 \checkmark \)
Constraints

Proper, redundant, or improper constraints

- Properly constrained.

\[ \sum F_y = -P + A + B + C = 0 \]
\[ \sum M_a = \vec{r}_p \times \vec{F} + \vec{r}_b \times \vec{B} + \vec{r}_c \times \vec{C} = 0 \]
\[ \sum M_c = \vec{r}_{sz} \times \vec{B} + \vec{r}_a \times \vec{A} + \vec{r}_{ps} \times \vec{P} \]

Redundancy? No.
3 unknowns
3 eqns. of equil.

You may use combinations of \( M \) equations about different points if less than 2 \( F \) equations are used.

Equations of Equilibrium options:
1) \( \sum F_x = 0 \)
2) \( \sum F_y = 0 \)
3) \( \sum M_a = 0 \)
4) \( \sum M_b = 0 \)
5) \( \sum M_c = 0 \).
Constraints

Proper, redundant, or improper constraints

Properly constrained

\[ \Sigma F_x = B_x = 0 \]
\[ \Sigma F_y = A + B_y - 200 \text{ lb} = 0 \]
\[ \Sigma M_B = r \times 200 \text{ lb} - r_A \times A = 0 \]

Redundant? No.
Constraints

Proper, redundant, or improper constraints

- Improperly constrained.

\[ \sum M_A = 2 \text{kN}\cdot\text{m} + \ell \cdot C \neq 0 \]
Determine the components of the support reactions at the fixed support A on the cantilevered beam.

\[ \Sigma F_x = A_x + (4 \text{ kN}) \cos 30^\circ = 0 \]
\[ \Sigma F_y = A_y - 6 \text{ kN} - (4 \text{ kN}) \sin 30^\circ = 0 \]
\[ \Sigma M_A = M - (1.5 \text{ m})(6 \text{ kN}) + \overrightarrow{r} \times \overrightarrow{F} = 0 \]