Announcements

- Quiz 2 next week ~

→ i-clicker out!

- Upcoming deadlines:
  - Friday (2/1) – Today!
    - WA#2
  - Tuesday (2/5)
    - PL HW

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Chapter 4: Force System Resultants
Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
Applications

Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force $F_H$ at the handle pull the nail? How can you mathematically model the effect of force $F_H$ at point $O$?

**Moment**

1. a very brief period of time. An Exact point in time. 2. importance. 3. A turning Effect produced by a force acting at a distance on An object.
Moment of a Force

All five forces shown above have the same magnitude, do they have the same effects on the wrench??
Moment of a Force

Which force(s) have NO turning effect?
Moment of a Force

Which force(s) yields a “tightly” effect?
Moment of a force – scalar formulation

The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

\[ M = d \cdot F \]

\[ F \perp d \]

(magnitude)

Direction: right hand rule.

\[ \text{Moment arm} \]

\[ \text{Moment} \]

\[ \text{Direction: right hand rule.} \]

\[ M_A \neq M_B \]

\[ Fd_A \neq Fd_B \]

Moments from a force always need the point about which the moment is calculated specified.
Example – Scalar Formulation

Determine the moment of this force about the point A as a function of F.

\[ M_A \neq r \times (F) \text{ since } r \text{ is not } \perp F \]

\[ \vec{F} = r_x \hat{i} + r_y \hat{j} \]

\[ \vec{F} = F_x \hat{i} + F_y \hat{j} \]

break down \( r \) & \( F \) into components & find corresponding \( \perp \) components instead.

\[ F_x = F \cos 36^\circ \]

\[ F_y = F \sin 36^\circ \]

\[ \vec{F} = F \hat{i} \]

\[ (r_x \hat{i}) \times (F \hat{i}) = r_x F \]

\[ (r_y \hat{j}) \times (F \hat{j}) = -r_y F \]

\[ M_A = \sum (r_x F_y \hat{k}) \]

\[ M_A = r_x F_y \hat{k} - r_y F_x \hat{k} \]

\[ M_A = \sum (r_x F_y - r_y F_x) \hat{k} \]

\[ \vec{M}_A = \sum (r_x F_y - r_y F_x) \hat{k} \]

Note: in 2D, direction of \( M \) is always \( \hat{k} \).

- clockwise: \(-\hat{k}\) (into the plane)
- counter-clockwise: \(+\hat{k}\) (out of the plane)
Moment of a force – vector formulation

The moment of a force \( \mathbf{F} \) about point \( \mathbf{O} \), or actually about the moment axis passing through \( \mathbf{O} \) and perpendicular to the plane containing \( \mathbf{O} \) and \( \mathbf{F} \), can be expressed using the cross (vector) product, namely:

\[
\vec{M} = \vec{r} \times \vec{F}
\]

\[
3D
\]

\[
\vec{M} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
F_x & F_y & F_z \\
r_x & r_y & r_z \\
\end{vmatrix}
= r_x F_y - r_y F_x \hat{k}
\]

\[
2D
\]

\[
\vec{M} = \begin{vmatrix}
\hat{i} & \hat{j} \\
F_x & F_y \\
\end{vmatrix}
= r_x F_y - r_y F_x \hat{k}
\]

\( \uparrow \) component = \( r_y (0) - (0) F_y = 0 \) in 2D, \( \vec{M} \) always \( \uparrow \).

\( \downarrow \) component = \( 0 (F_x) - r_x (0) = 0 \) direction is always \( \uparrow \).
Example – Vector Formulation

Given: The angle $\theta = 30^\circ$ and $x = 10$ m.

Find: The moment by $\vec{P}$ about point $O$.

$$\vec{M}_o = \vec{r}_{OA} \times \vec{P}$$

$$\vec{P} = P\hat{u}_{AB}, \quad \hat{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{\vec{r}_B - \vec{r}_A}{r_{AB}}$$