Printout

Friday, February 1, 2019 10:41 AM

Announcements

- Quiz 2 next week ~

→ i-clicker out!

❑ Upcoming deadlines:

- Friday (2/1) – Today!
  - WA#2
- Tuesday (2/5)
  - PL HW

Happy

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Chapter 4: Force System Resultants

Goals and Objectives

- Discuss the concept of the moment of a force and show how to calculate it in two and three dimensions

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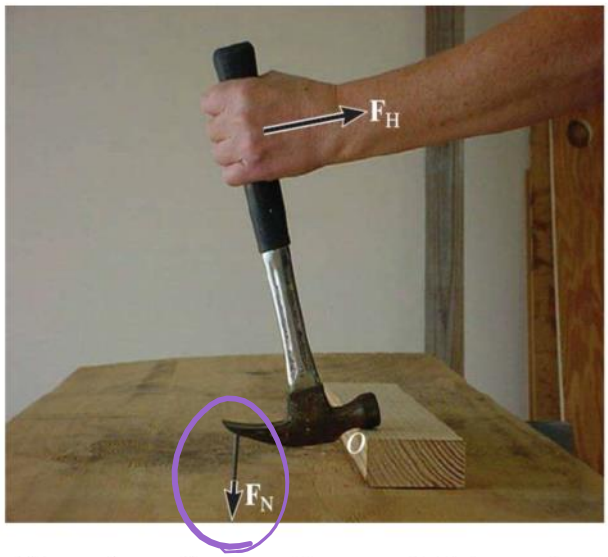
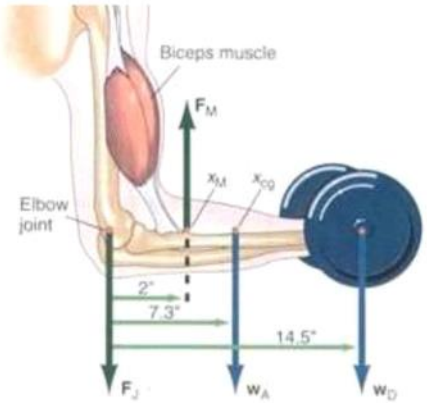
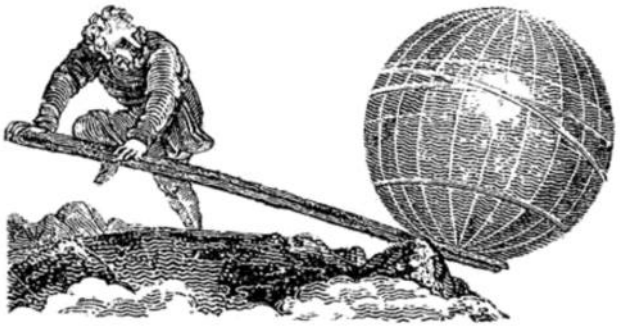
Applications



Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?





Moment 1.a very brief period of time. An Exact point in time. 2. importance. 3. A turning Effect produced by a force acting at a distance on An object.

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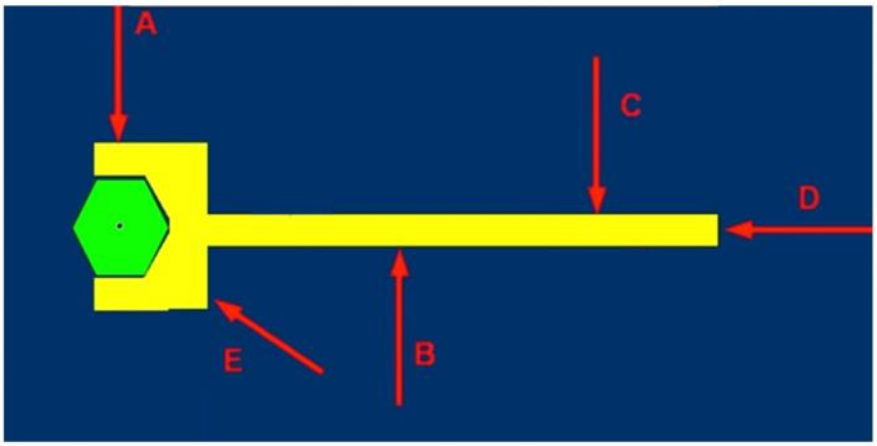
Moment of a Force



All five forces shown above have the same magnitude, do they have the same effects on the wrench??

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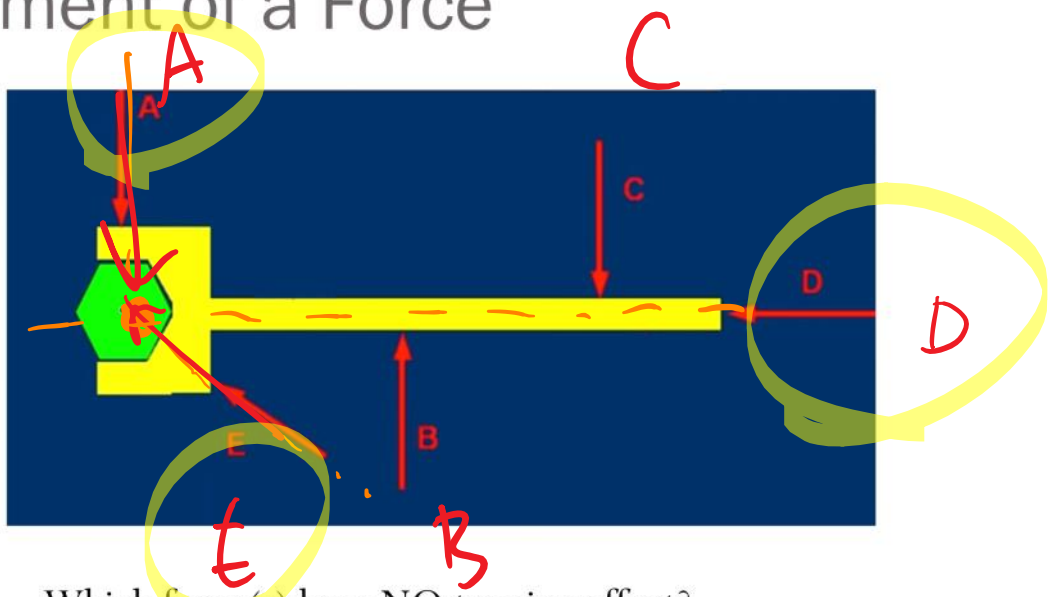
Moment of a Force



Which force(s) have NO turning effect?

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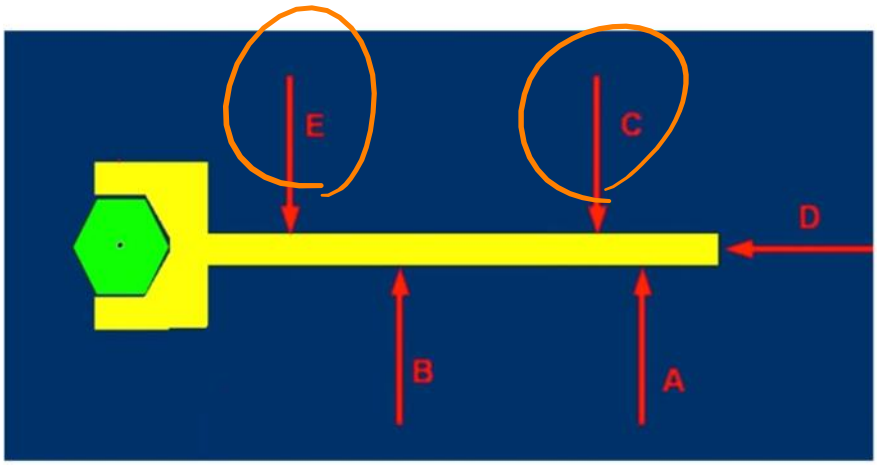
Moment of a Force



Which force(s) yields a "tighty" effect?

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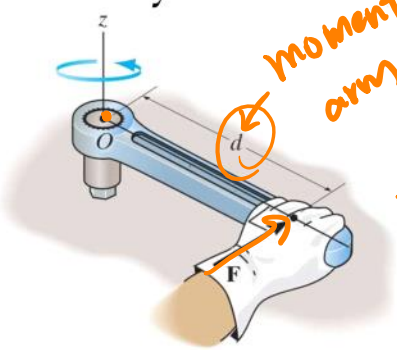
Moment of a force – scalar formulation

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).



moment arm

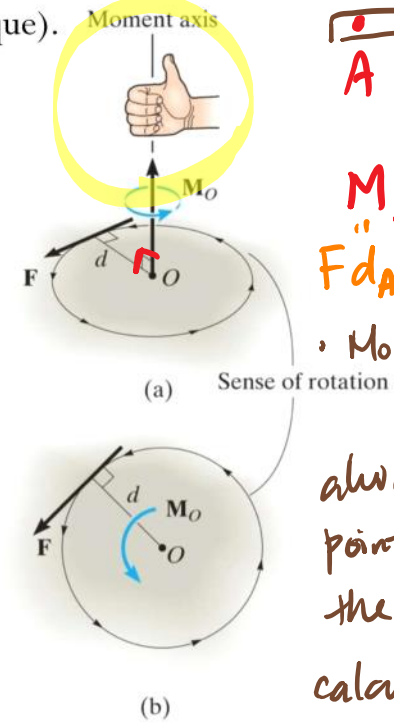
$M = d \cdot F$

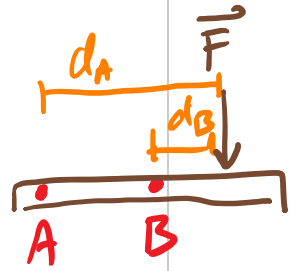
$\rightarrow F \perp d$

(magnitude)

Direction: right hand rule.

d F M





$M_A \neq M_B$

$F d_A$ $F d_B$

• Moment from a force always needs the point about which the moment is calculated specified.

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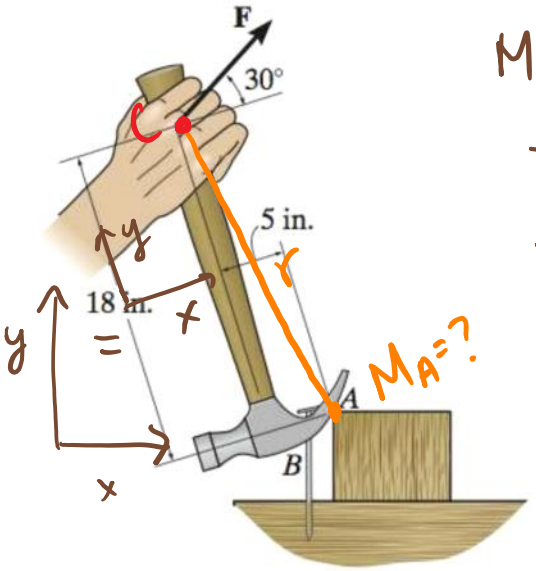
Example – Scalar Formulation

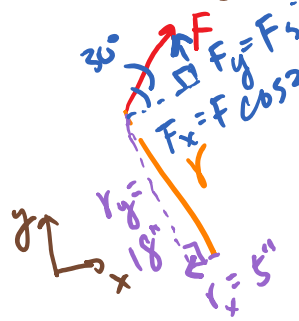
Determine the moment of this force about the point A as a function of $\mathbf{F}$.

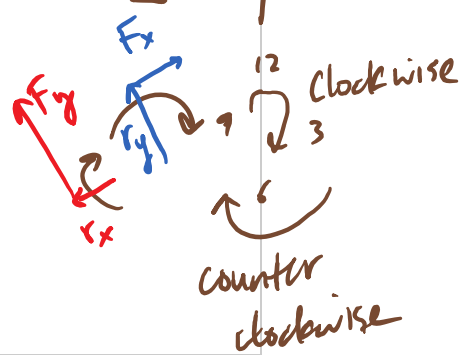


$M_A \neq (r)(F)$ since r is not $\perp$ F

$\vec{r} = r_x\hat{i} + r_y\hat{j}$) break down r & F

$\vec{F} = F_x\hat{i} + F_y\hat{j}$ } into components & find corresponding $\perp$ components instead.





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$$M_A = \sum \begin{cases} M_{A1} = r_x F_y \quad (-\hat{k}) \\ M_{A2} = r_y F_x \quad (-\hat{k}) \end{cases} \rightarrow \vec{M}_A = (-r_x F_y - r_y F_x)\hat{k}$$

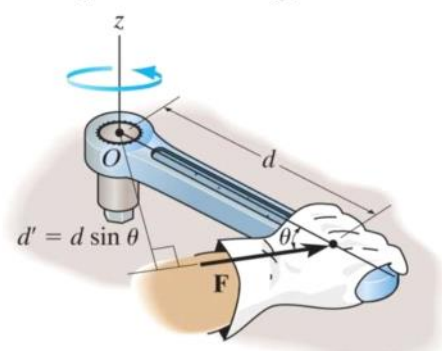
- Note: in 2D, direction of M is always $\hat{k}$.
  - ~ clockwise = $-\hat{k}$ (into the plane)
  - ~ counter-clockwise = $+\hat{k}$ (out of the plane)

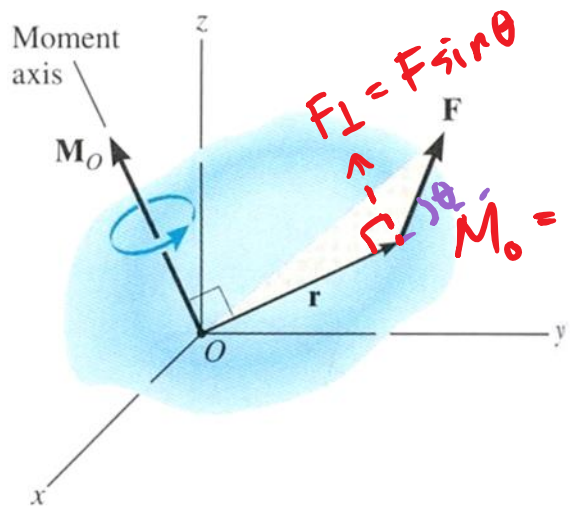
Moment of a force – vector formulation

The moment of a force **F** about point **O**, or actually about the moment axis passing through O and perpendicular to the plane containing **O** and **F**, can be expressed using the cross (vector) product, namely:

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{r} \times \vec{F} = |r||F|\sin\theta$$





$F_\perp = F\sin\theta$

$M_o = r \cdot F_\perp$

3D

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

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2D

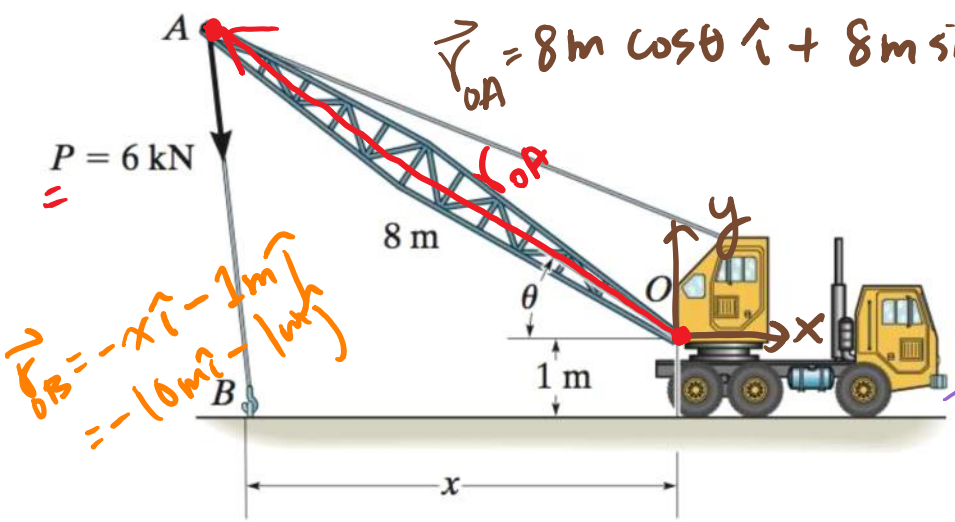
$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & 0 \\ F_x & F_y & 0 \end{vmatrix} = r_x F_y - r_y F_x \, \hat{k}$$

$\hat{i}$ component $= r_y(0) - (0)F_y = 0$

$\hat{j}$ component $= 0(F_x) - r_x(0) = 0$

in 2D, $\vec{M}$ direction is always $\pm\hat{k}$.

Example – Vector Formulation



$\vec{r}_{OA}$ in this case

P for this problem

Given: The angle $\theta = 30°$ and $x = 10$ m.

Find: The moment by **P** about point O.

Eqn: $\vec{M}_O = \vec{r} \times \vec{F}$

point of interest.

$\vec{M}_O = \vec{r}_{OA} \times \vec{P}$

$\vec{P} = P\hat{u}_{AB}, \quad \hat{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{\vec{r}_{OB} - \vec{r}_{OA}}{r_{AB}}$

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