Goals and Objectives

- Example problems for particle(s) at equilibrium in 2D and 3D.
Example – Single Particle in 3D

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 360 \text{ N-m}$.

Plan: Use ring O to relate all the forces involved (known & unknown) to each other.

**Equations of Equilibrium (EoE)**

\[ \Sigma F_x = F_{Ax} + F_{Bx} + F_{Cx} = 0 \]
\[ \Sigma F_y = F_{Ay} + F_{By} + F_{Cy} = 0 \]
\[ \Sigma F_z = F_{Az} + F_{Bz} + F_{Cz} - W = 0 \]

Find the Cartesian components of each force through the green figure:

\[ \vec{F}_A = F_{Ax} \hat{i} + F_{Ay} \hat{j} + F_{Az} \hat{k} = \vec{O} - k s_A \hat{j} + k \hat{k} = 0 \]
\[ \vec{F}_B = F_{Bx} \hat{i} + F_{By} \hat{j} + F_{Bz} \hat{k} = -k s_B + D \hat{j} + 0 \hat{k} = 0 \]
\[ \vec{F}_C = F_{Cx} \hat{i} + F_{Cy} \hat{j} + F_{Cz} \hat{k} = \vec{F}_c \hat{i} = \hat{F}_c \left( \frac{4}{14} \right) + F_c \left( \frac{4}{14} \right) \hat{j} + F_c \left( \frac{12}{14} \right) \hat{k} \]

\[ \hat{u}_c = \frac{6i + 4j + 12k}{\sqrt{6^2 + 4^2 + 12^2}} = \frac{6}{14} \hat{i} + \frac{4}{14} \hat{j} + \frac{12}{14} \hat{k} \]

EoE becomes

\[ \Sigma F_x = 0 + (-k s_B) + F_c \left( \frac{6}{14} \right) = 0 \quad F_c \approx 229 \text{ N} \]
\[ \Sigma F_y = (-k s_A) + 0 + F_c \left( \frac{4}{14} \right) = 0 \quad s_A \approx 0.182 \text{ m} \]
\[ \Sigma F_z = 0 + 0 + F_c \left( \frac{12}{14} \right) - W = 0 \quad s_B \approx 0.273 \text{ m} \]
Example – System of Particles

The five ropes can each take 1500 N without breaking. How heavy can $W$ be without breaking any?

Plan: See which cable will experience the most force by relating them to each other via FBD, pick 1 as reference for comparison (e.g. $T_1$)

1. **FBD for A**
   - $F_{\Sigma y} = 2T_1 - T_2 = 0$
   - $T_2 = 2T_1$

2. **FBD for B**
   - $F_{\Sigma y} = T_3 - 2T_2 = 0$
   - $T_3 = 2T_2 = 4T_1$

3. **FBD for W**
   - $F_{\Sigma y} = T_2 + T_4 + T_5 - W = 0$
   - $W = T_2 + T_4 + T_5$
   - $= 2T_1 + \frac{1}{2}T_1 + T_1$
   - $= \frac{3}{2}T_1$

4. **FBD for C**
   - $F_{\Sigma y} = T_4 - 2T_4 = 0$
   - $T_4 = \frac{1}{2}T_1$

5. **FBD for D**
   - $F_{\Sigma y} = 2T_4 - T_5 = 0$
   - $T_5 = 2T_4 = T_1$

*Since $T_2 (= 4T_1) > T_2 (= 2T_1) > T_5 (= T_1) > T_4 (= \frac{1}{2}T_1)$,*
$T_3$ experiences the highest load, so it is the limiting factor.

Let $T_3 = 1500 \text{ N}$, $4T_1 = 4 \left( \frac{W}{4} \right)$, the maximum $W = 1312.5 \text{ N}$.
Example – System of Particles

The 30-kg bucket is supported at $E$ by a system of five cords. Determine the force in each cord for equilibrium.

There are 5 cords with unknown force magnitudes (but unknown direction based on the image), 5 equations are needed to solve for them. Each FBD can produce 2 EoE ($\Sigma F_x = 0$ & $\Sigma F_y = 0$), at least 3 FBDs are necessary to solve the problem.

First use FBD for the bucket:

Second, FBD for ring E:

Third, FBD for B:

(its the same with $F_{BE} = F_{EB}$ solved from the previous part, now solve for $F_{BA}$ and $F_{AB}$.)
Example – System of Particles

Determine the tension in each cable for the system below, given that the tension in cable \( AB \) is 10 N.

In 3D, 3 equations of equilibrium are available for each FBD \((\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0)\), 3 unknowns can be solved with each FBD. There are 5 unknowns here \((F_{BA}, F_{BD}, F_{BE}, F_{EF}, F_{EG})\), at least 2 FBD are needed to solve.

\[\begin{align*}
\sum F_x &= F_{BAx} + F_{BEx} + F_{BDx} + F_{DEx} = 0 \\
\sum F_y &= F_{BAy} + F_{BEy} + F_{BDy} + F_{BEy} = 0 \\
\sum F_z &= F_{BAz} + F_{B Ez} + F_{BDz} + F_{BEz} = 0
\end{align*}\]

- Use position vector to derive force directions. For example: \( \vec{F}_{BA} = F_{BA} \hat{u}_{BA} = F_{BA} \left( \frac{\vec{r}_{BA}}{r_{BA}} \right) \)
  \[\hat{u}_{BA} = \frac{1.4\hat{i} + 0\hat{j} + 0\hat{k}}{1.4} = \hat{i}, \quad \text{so} \quad \vec{F}_{BA} = F_{BA} \hat{i}\]

\[\rightarrow \quad \text{Once } F_{BA}, F_{BD}, \text{ and } F_{BE} \text{ are solved from above, then}\]

\[\begin{align*}
\sum F_x &= F_{BAx} + F_{BEx} + F_{BDx} + F_{EGx} + F_{EFx} = 0 \\
\sum F_y &= F_{BAy} + F_{BEy} + F_{BDy} + F_{EGy} + F_{EFy} = 0 \\
\sum F_z &= F_{BAz} + F_{B Ez} + F_{BDz} + F_{EGz} + F_{EFz} = 0
\end{align*}\]

\[\begin{align*}
\sum F_e &= F_{BAx} + F_{BEx} + F_{BDx} + F_{EGx} + F_{EFx} = 0 \\
\sum F_y &= F_{BAy} + F_{BEy} + F_{BDy} + F_{EGy} + F_{EFy} = 0 \\
\sum F_z &= F_{BAz} + F_{B Ez} + F_{BDz} + F_{EGz} + F_{EFz} = 0
\end{align*}\]
\[ F_x = F_{ex} + F_{fx} + F_{ex} = 0 \]
\[ F_y = F_{ey} + F_{fy} + F_{ey} = 0 \]