Announcements

- Quiz 1 starts tomorrow! (Thursday, January 24)
- CATME survey available now

- Upcoming deadlines:
- Friday (01/25)
  - Written Assignment 1
- Tuesday (01/29)
  - Prairie Learn HW2

Lecture Objectives

- Vector Dot Product
- Vector Projection
- Vector Cross Product

Dot (or scalar) product

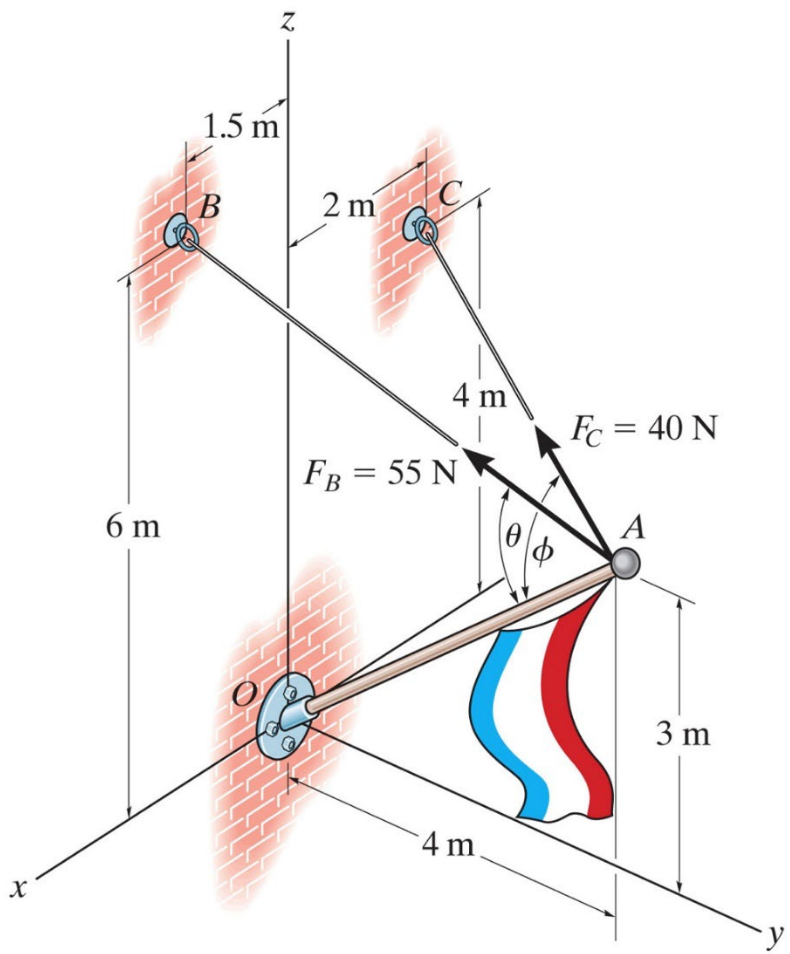
The dot product of vectors **A** and **B** is defined as such

Cartesian vector formulation:

$$\boldsymbol{A} \cdot \boldsymbol{B} \quad =$$

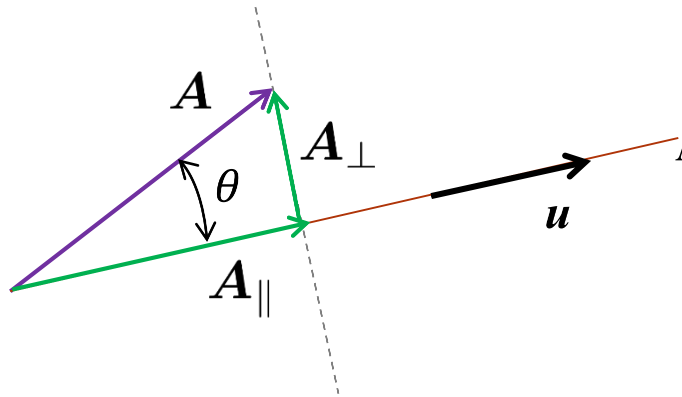
Example

Determine the angle between *AB* and the axis *AO* of the flag pole.



Projections

The scalar component $A_{||}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:



$$A_{||} =$$

And thus the <u>vector</u> components $\boldsymbol{A}_{||}$ and $\boldsymbol{A}_\perp$ are given by:

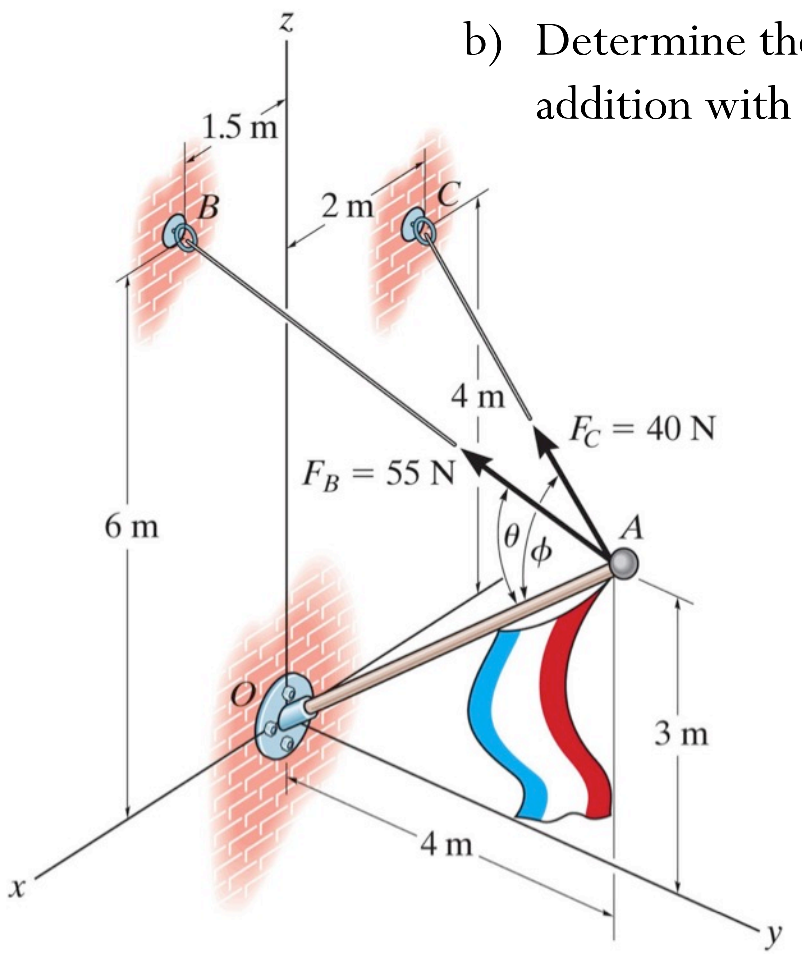
$$\boldsymbol{A}_{||} =$$

$$\boldsymbol{A}_\perp =$$

Example

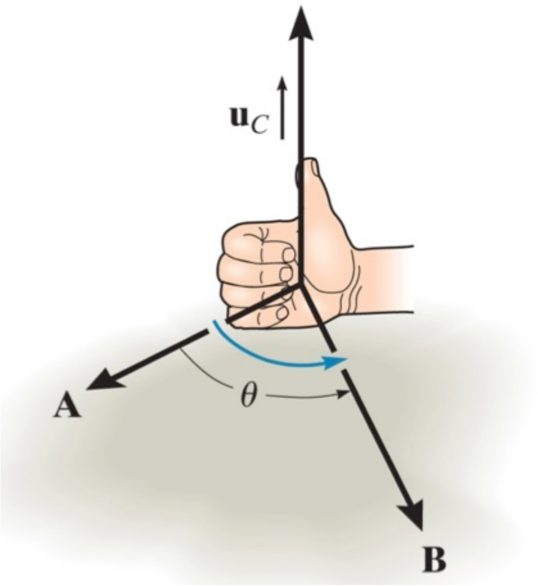
a) Determine the projected component of the force vector $\boldsymbol{F}_C$ along the axis AO of the flag pole.

b) Determine the perpendicular component such at its vector addition with the projected component equals $\boldsymbol{F}_C$.



Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



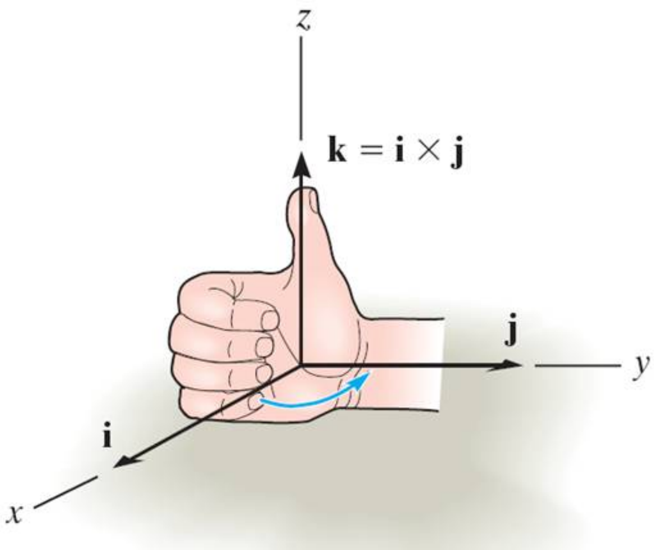
$$\boxed{\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}}$$

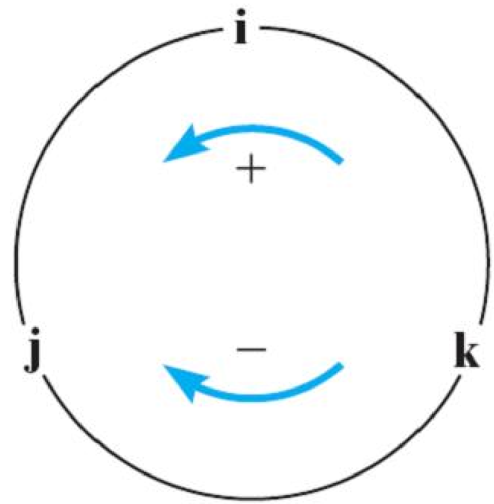
The magnitude of vector **C** is given by:

The vector **C** is perpendicular to the plane containing **A** and **B** (specified by the **right-hand rule**). Hence,

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





Considering the cross product in Cartesian coordinates

$$\begin{aligned}
\boldsymbol{A} \times \boldsymbol{B} &= (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \times (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) \\
&= +A_xB_x(\boldsymbol{i} \times \boldsymbol{i}) + A_xB_y(\boldsymbol{i} \times \boldsymbol{j}) + A_xB_z(\boldsymbol{i} \times \boldsymbol{k}) \\
&\quad +A_yB_x(\boldsymbol{j} \times \boldsymbol{i}) + A_yB_y(\boldsymbol{j} \times \boldsymbol{j}) + A_yB_z(\boldsymbol{j} \times \boldsymbol{k}) \\
&\quad +A_zB_x(\boldsymbol{k} \times \boldsymbol{i}) + A_zB_y(\boldsymbol{k} \times \boldsymbol{j}) + A_zB_z(\boldsymbol{k} \times \boldsymbol{k}) \\
&= (A_yB_z - A_zB_y)\boldsymbol{i} - (A_xB_z - A_zB_x)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}
\end{aligned}$$

Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2×2 determinants.

Example

Determine the area of the parallelogram spanned by the vectors $\mathbf{a} = (3, -3, 1)$ and $\mathbf{b} = (4, 9, 2)$.