Virtual disp = \( \delta x, \delta y, \delta \theta \)

Virtual work = \( \delta V \)

\( \sum \delta V = 0! \)
- As system becomes more complicated, the # of mech. based eqns increases a lot!
- Sometimes it can't be solved b/c of too many unknowns
- Virt work can solve some of these more easily.
- We will focus on systems w/ one degree of freedom
  \[ \vec{\delta} \theta \rightarrow \delta \theta \rightarrow \text{the } \delta \text{ in } \theta \text{ will can be } \delta \text{ used to determine the nature of describe the } \delta \text{ virtual displacement } \]
Procedure

1. Draw FBD + find $\theta$
2. Sketch deflected position assuming a positive $\theta$
3. Remove forces that do not contribute to work

4. Indicate position coordinate system from a fixed point $(x, y)$

5. They should be parallel to force line of action

6. Identify virtual displacements using #4

7. Solve $S_x + S_y$ in terms of $\delta \theta$
7) Write virtual work eqns for each virtual displacement.

8) Factor out displacements/other constants & solve for force/moments.
1. Draw FBD and find $\theta$

2. Sketch deflected position assuming a positive virtual displacement.
3. Remove forces that do not contribute to work.

4. Indicate position coordinate system from a fixed point.

If A moves to the right, \( \Theta \), \( \Rightarrow \) positive direction.

If A moves to the left, \( \Theta \), \( \Rightarrow \) negative direction.
5. Identify virtual displacements

4. Solve $\delta x$ and $\delta y$ in terms of $\delta \Theta$.
   - Careful with coordinate system!

\[ X_A = L \cos \Theta \]
\[ \delta X_A = -L \sin \Theta \delta \Theta \]
\[ Y_W = \frac{1}{2} L \sin \Theta \]
\[ \delta Y_W = \frac{1}{2} L \cos \Theta \delta \Theta \]
Write virtual work eqns for each virtual displacement assuming the positive displacement.

\[ \delta U = P \delta x_A + W \delta y_w = 0 \]

\[ \delta x_A = -L \sin \theta \delta \theta \]

\[ \delta y_w = \frac{1}{2} L \cos \theta \delta \theta \]

\[ -P (-L \sin \theta \delta \theta) + (-W) \left( \frac{1}{2} L \cos \theta \delta \theta \right) = 0 \]

\[ PL \sin \theta \delta \theta - \frac{1}{2} WL \cos \theta \delta \theta = 0 \]
Look! What if we change the coordinate system?

\[ x_A = -L \cos \theta \]
\[ S_{x_A} = L \sin \theta \cos \theta \]
\[ y_w = \frac{1}{2} L \sin \theta \]
\[ S_{y_w} = \frac{1}{2} L \cos \theta \cos \theta \]

\[ U = PS_{x_A} + W S_{y_w} = 0 \]
\[ = PL \sin \theta \cos \theta - W \frac{1}{2} L \cos \theta \cos \theta = 0 \]
\[ = P \sin \theta - \frac{1}{2} WL \cos \theta = 0! \]

Yahoo! Same answer!
What if we choose a different deflection?

2

3

4

\[ P \rightarrow 2\theta \]

\[ N_B \]

\[ W \]

\[ L \]

\[ x \]
\( X_A = -L \cos \theta \)
\( Y_w = \frac{1}{2} L \sin \theta \)
\( S_{X_A} = L \sin \theta \cos \theta \)
\( S_{Y_w} = \frac{1}{2} L \cos \theta \sin \theta \)

\[ EU = P S_{X_A} + W S_{Y_w} \]
\[ = PL \sin \theta \cos \theta - W \left( \frac{1}{2} L \cos \theta \sin \theta \right) = 0 \]
\[ = Ps \sin \theta - \frac{1}{2} W \cos \theta = 0 \]

Nice! We win again!
(8) Factor out displacements/other constants and solve for force/moments

\[ L \sin \theta \left( P \sin \theta - \frac{1}{2} W \cos \theta \right) = 0 \]

\[ P \sin \theta = \frac{1}{2} W \cos \theta \]

\[ P = \frac{1}{2} W \frac{1}{\tan \theta} \]
1. Draw FBD and find $\theta$

2. Sketch deflected position assuming a positive $8\theta$
3. Remove forces that do not contribute to work

4. Indicate position coordinate system from a fixed point
5. Identify virtual displacements

6. Solve $\delta x$, $\delta y$ in terms of $\delta \theta$

$y_{w,1} = \frac{1}{2} L \sin \theta$

$S_{yw,1} = \frac{1}{2} L \cos \theta \delta \theta$

$x_B = 2L \cos \theta$

$\delta x_B = -2L \sin \theta \delta \theta$
Write virtual work eqns

\[ \delta y_{w,1} = \frac{1}{2} L \cos \theta \delta \theta \quad \delta x_B = -2L \sin \theta \delta \theta \]

\[ \sum \delta y: W_1 \delta y_1 + W_2 \delta y_2 + FSx_B = 0 \]

\[ W_1 = W_2 \quad 2W \delta y_2 + FSx_B = 0 \]

\[ 2W\left(\frac{1}{2} L \cos \theta \delta \theta\right) + F(-2L \sin \theta \delta \theta) = 0 \]

\[ (WL \cos \theta - 2FL \sin \theta) \delta \theta = 0 \]

\[ W \cos \theta = 2F \sin \theta \]

\[ \frac{W}{2F} = \tan \theta \quad \theta = \tan^{-1}\left(\frac{W}{2F}\right) \]
What if it deflected to the right?

Virtual displacements