Housekeeping

- Today
  - Extra credit due (see Piazza)
- Tuesday
  - PL HW 6
  - First quiz day
- Thursday
  - ME HW 7
- Saturday
  - Last quiz day
- Sunday
  - WA due
**Moment of a force – scalar formulation**

The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).

**Moment of a force – vector formulation**

The **moment of a force about a point** provides a measure of the **tendency for rotation**.

The moment of a force \( \vec{F} \) about point \( O \), or actually about the moment axis passing through \( O \) and perpendicular to the plane containing \( O \) and \( \vec{F} \), can be expressed using the cross (vector) product, namely:

\[
\vec{M}_o = \vec{r}_o \times \vec{F}
\]

**Position vector from \( O \) to any point along line of action of \( \vec{F} \)**

**Scalar magnitude**

\[
|\vec{M}| = |\vec{r}| |\vec{F}| \sin \theta
\]

**Vector**

\[
\vec{M} = |\vec{r}| |\vec{F}| \sin \theta \quad (\vec{r}_o \times \vec{F})
\]
Moment of a force about a specified axis

\[ \mathbf{M}_O = \mathbf{r} \times \mathbf{F} \]
Moment of a force about specified axis
Determine the moment of the force $F$ about the axis extending between $A$ and $C$.

The force $F = \{4i + 12j - 3k\}$ lb acts through point $B$. The diagram shows the dimensions $4\text{ ft}$, $3\text{ ft}$, and $2\text{ ft}$.
Couples ...
Moment of a couple
To hipster or not to hipster...
Determine the magnitude and coordinate direction angles of the couple moment. The pipe line assembly lies in the x-y plane. Assume $F = 80\, \text{N}$. 

![Pipe line assembly diagram]
Determine the couple moment acting on the pipe
Moving a force on its line of action

Moving a force from A to B, when both points are on the vector’s line of action, does not change the external effect.

Hence, a force vector is called a sliding vector.

However, the internal effect of the force on the body does depend on where the force is applied.
Moving a force off of its line of action

\[ M = Fd \]
Given: A 450 N force couple acting on the pipe assembly.

Find: The couple moment in Cartesian vector notation.
Given: Two couples act on the beam with the geometry shown and $d = 4$ ft.

Find: The resultant couple
Equipollent (or equivalent) force systems

A force **system** is a collection of **forces** and **couples** applied to a body.

Two force systems are said to be **equipollent** (or equivalent) if they have the **same resultant force** AND the **same resultant moment** with respect to any point \( P \).
What is the equivalent system?
Special cases of equivalent systems

If $F_R$ perpendicular to $(M_R)_0$ and $F_R \neq 0$, then an equivalent system consisting of ONLY a single force can always be found. There are three possibilities:

1) Concurrent Force System
The lines of action of all the forces intersects at a common point $O$.

![Diagram of concurrent force system](image)
2) Coplanar Force System

How can we replace this force system by an equivalent force \( \mathbf{F}_R \) and a couple moment about point \( O \) \( (\mathbf{M}_R)_O \)?

(a) \hspace{2cm} (b)
3) Parallel Force System

\[ \text{Parallel Force System} \]

\[ F_1 + F_2 + F_3 = 0 \]

\[ \text{Moments Coupless} \]

\[ L_9 - \text{Moments Coupless} \]

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