Housekeeping

- Practice quiz available on PL website
- Thursday
  - Mastering engineering HW3
- Friday (5pm)
  - Last quiz time
- Sunday
  - WA2

Get your i-Clickers out!

The Best Female Rock Climber In the World is 14 Years Old

You tube: crane accidents caught on tape
Chandeliers swaying on 38th floor of Denver hotel as storm passes
Recap

• Position and unit vectors

\[ \mathbf{r}_{AB} = (\text{end}-\text{start}) \hat{i}, \hat{j}, \hat{k} \]

• Dot (scalar) product

- How much of a force acts in a given direction

- Calculate the dot product between 2 vectors

\[ \mathbf{A} \cdot \mathbf{B} = \cos \theta \frac{||A||B||}{A||B||} \]
Vector algebra, as we are going to use it, is based on a ______ coordinate system.

A) Euclidean
B) Left-handed
C) Greek
D) Right-handed
E) Egyptian
Given points A and B, determine the position vector \( \vec{r}_{AB} \).

**Coordinates**

- \( A = (10, 10, 200) \) cm
- \( B = (50, 50, 0) \) cm

**Position vector** \( \vec{r}_{AB} \):

- \( A \): \( <60, 60, 200> \) cm
- \( B \): \( <-40, -40, 200> \) cm
- \( C \): \( <40, 40, -200> \) cm
- \( D \): \( <-60, -60, -200> \) cm
Example

Which cartesian components of force exist in strut AO?

(A) i and j

(B) j and k

(C) i and k

(D) i, j, and k
Example

Determine the projected component of the force vector $F_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector.
How much of the project of \( \vec{F}_{AC} \) is along \( \vec{AO} \)?

\[ (\vec{F}_{AC} \cdot \vec{u}_{AO}) \]

\[ \vec{F}_{AC} = F_{AC} \vec{u}_{AC} \quad (1) \]

\[ \vec{u}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} \quad \text{need } \vec{r}_{AC} \]

\[ \vec{r}_{AC} = \left< 5 \cos 60^\circ, -6, (5 \sin 60^\circ - 2) \right> \text{ ft} \]

\[ |\vec{r}_{AC}| = \text{norm}(\vec{r}_{AC}) \]

\[ \vec{u}_{AC} = \left< 0.362, -0.8689, 0.3375 \right> \]

From (1)

\[ \vec{F}_{AC} = F_{AC} \vec{u}_{AC} = \left< 21.72, -52.14, 20.25 \right> \text{ lb} \]

\[ \vec{u}_{AO} = \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} \]

\[ \vec{r}_{AO} = \left< 0, -6, -2 \right> \text{ ft} \]

\[ |\vec{r}_{AO}| = \text{radius} \]

\[ \vec{u}_{AO} = \left< 0, -0.9987, -0.3162 \right> \]

See slide 6

\[ \vec{F} = F \vec{u} \]

In previous example we were given this, this time we need to calculate it.
A: \((\vec{F}_{Ac} \cdot \vec{U}_{A0}) \vec{U}_{A0}\)

A: 43.06 \langle 0, -0.95, -0.32 \rangle \text{ lb}
Cross (or vector) product

The cross product of vectors $\mathbf{A}$ and $\mathbf{B}$ yields the vector $\mathbf{C}$, which is written as:

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

The magnitude of vector $\mathbf{C}$ is given by:

$$|\mathbf{C}| = |\mathbf{A}| |\mathbf{B}| \sin \theta$$

The vector $\mathbf{C}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ (specified by the right-hand rule). Hence,

$$\mathbf{C} = AB \sin \theta \, \hat{u}_c$$

$$\mathbf{B} \times \mathbf{A} = -\mathbf{C}$$

$$\mathbf{A} \times -\mathbf{C} = \mathbf{B}$$
The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$

Considering the cross product in Cartesian coordinates:

$$\mathbf{A} \times \mathbf{B} = (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \times (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \mathbf{i} - (A_x B_z - A_z B_x) \mathbf{j} + (A_x B_y - A_y B_x) \mathbf{k}$$
Also, the cross product can be written as a determinant.

\[
A \times B = \begin{vmatrix}
  \mathbf{i} & \mathbf{j} & \mathbf{k} \\
  A_x & A_y & A_z \\
  B_x & B_y & B_z 
\end{vmatrix}
\]

Each component can be determined using \(2 \times 2\) determinants.

\[
\begin{vmatrix}
  \hat{i} & \hat{j} & \hat{k} \\
  A_x & A_y & A_z \\
  B_x & B_y & B_z 
\end{vmatrix}
= \begin{vmatrix}
  A_x & A_y & A_z \\
  B_x & B_y & B_z 
\end{vmatrix} + \begin{vmatrix}
  A_x & A_y & A_z \\
  B_x & B_y & B_z 
\end{vmatrix}
\]

(see previous slide)
Chap 2 - recap

- Scalars – magnitudes (e.g. positions, forces, displacements)
- Vectors – magnitude + direction (how much force in a given direction (projection)
- Dot product – force between 2 other vectors (moments)
- Cross product – force along a line (Cross product)

Rectangular components of a vector
Cartesian vector representation
Cartesian vector using direction cosines
Cartesian vector using unit vector
Cartesian position vectors
Chapter 3: Equilibrium of a particle
For a spool of given weight, how would you find the forces in cables AB and AC? If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn’t fail.
Equilibrium of a particle

According to Newton’s first law of motion, a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

\[ \sum F = 0 \Rightarrow \vec{a} = \vec{0} \]

where \( \sum F = 0 \) is the resultant force vector of all forces acting on a particle.

In three dimensions, equilibrium requires:

\[ \sum \vec{F} = \sum \vec{F}_x + \sum \vec{F}_y + \sum \vec{F}_z = \vec{0} \]

- Important to know which forces are acting on the system
Equilibrium of a particle (cont.)

Contact force in smooth surface:

\[ \theta_1 \quad \theta_2 \]

uniform objects of weight \( W \)

\[ \vec{F}_1 \quad \vec{F}_2 \]

FBD

\[ \vec{F}_1 \quad \vec{F}_2 \]

\[ W \]
Typical Q: Find the tension in the cables for a given weight

Step 1: Read the Q and understand what is being asked

Free body diagram

Model of IRL scenario
1. Draw particle(s) or rigid body(ies)
2. Draw forces acting on (1)
3. Indicate any geometry, lengths
4. Draw a coordinate system