Housekeeping

- Practice quiz available on PL website
- Tuesday
  - Prairie Learn HW2
  - First day for QUIZ (short session because of holiday)
- Thursday
  - Mastering engineering HW3
- Friday (5pm)
  - Last quiz time
- Sunday
  - WA2
Recap

• Cartesian vectors

• Unit vectors

• Resultant forces
<clicker time>
Position vectors

A position vector \( \mathbf{r} \) is defined as a fixed vector which locates a point in space relative to another point. For example,

\[
\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}
\]

expresses the position of point \( P(x,y,z) \) with respect to the origin \( O \).

The position vector \( \mathbf{r} \) of point \( B \) with respect to point \( A \) is obtained from

\[
\mathbf{r}_B = \mathbf{r} + \mathbf{r}_A
\]

\[
\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A
\]

\[
\mathbf{r} = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})
\]

\[
\mathbf{r} = (x_B-x_A)\mathbf{i} + (y_B-y_A)\mathbf{j} + (z_B-z_A)\mathbf{k}
\]
Example

The ring at D is midway between points A and B. Determine the lengths of wires AD, BD and CD.
The force vector \( F \) acting along the rope can be defined by the unit vector \( u \) (defined the direction of the rope) and the magnitude of the force.

\[
F = F \, u
\]

The unit vector \( u \) is specified by the position vector:

The man pulls on the cord with a force of 70 lb. Represent the force \( F \) as a Cartesian vector.
Dot (or scalar) product

The dot product of vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as such

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos(\theta)$$

Laws of operation:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

$$\alpha(\mathbf{A} \cdot \mathbf{B}) = \alpha \mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \alpha \mathbf{B}$$

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$$

Cartesian vector formulation:

$$\mathbf{A} \cdot \mathbf{B} = (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \cdot (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k})$$

Note that:

$$\mathbf{i} \cdot \mathbf{j} = 0 \quad \mathbf{i} \cdot \mathbf{i} = 1$$
Example

Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.
Example

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L4 - Position Vectors Force along a line Cross product
Example

Determine the projected component of the force vector $F_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector.

![Diagram](image.png)

- $F_{AB} = 70 \text{ lb}$
- $F_{AC} = 60 \text{ lb}$
Vectors come in many packages

Rectangular components of a vector

Cartesian vector representation

Cartesian vector using direction cosines

Cartesian vector using unit vector

Cartesian position vectors
i>clicker time
Cross (or vector) product

The cross product of vectors $\mathbf{A}$ and $\mathbf{B}$ yields the vector $\mathbf{C}$, which is written

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

The magnitude of vector $\mathbf{C}$ is given by:

The vector $\mathbf{C}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$ (specified by the right-hand rule). Hence,

$$\mathbf{C} = .$$
Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$

Considering the cross product in Cartesian coordinates

$$A \times B = (A_x i + A_y j + A_z k) \times (B_x i + B_y j + B_z k)$$
Cross (or vector) product

Also, the cross product can be written as a determinant.

\[
\mathbf{A} \times \mathbf{B} = \begin{vmatrix}
i & j & k \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

Each component can be determined using $2 \times 2$ determinants.
Chap 2 - recap

- Scalars –
- Vectors –
- Dot product –
- Cross product –
Chapter 3: Equilibrium of a particle
Applications

For a spool of given weight, how would you find the forces in cables AB and AC? If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn’t fail.
Equilibrium of a particle

According to Newton’s first law of motion, a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\sum F = 0$$

where $\sum F = 0$ is the resultant force vector of all forces acting on a particle.

In three dimensions, equilibrium requires:
Free body diagram
Equilibrium of a particle (cont.)

Contact force in smooth surface:

\[ \theta_1 \]

\[ \theta_2 \]
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