Housekeeping

- Practice quiz available on PL website
- TODAY!
  - Prairie Learn HW0
- Sunday
  - Written assignment 1
  - Mastering engineering HW1
  - Excused absence form (see Policies on website) can be used for issues due to late registration, lack of access (send a screen shot)
- Tuesday
  - Prairie Learn HW2
  - First day for QUIZ (short session because of holiday)
- Thursday
  - Mastering engineering HW3
- Friday (5pm)
  - Last quiz time

Brace yourselves
Recap

- Pay attention to units!
- Solve problem symbolically
- Equations must be dimensionally homogenous
- 1% accuracy
- Scalar – defined by magnitude (negative/positive)
- Vector – defined by magnitude and direction
- Vector operations – addition/subtraction
1. Which one of the following is a scalar quantity?

A) Speed

B) Displacement

C) Weight

D) Momentum

E) ha ha
Cartesian vectors

Rectangular coordinate system: formed by 3 mutually perpendicular axes, the $x$, $y$, $z$ axes, with unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ in these directions.

We use the “$\wedge$” to identify basis vectors (instead of the “$\sim$” notation)
Given $\vec{F}_1$ and $\vec{F}_2$

Q: Solve for the resultant force and its direction

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

$$= 3F_x \uparrow + 3F_y \uparrow$$

$$= (F_{1,x} - F_{2,x}) \uparrow + (F_{1,y} + F_{2,y}) \uparrow$$

$$\vec{F}_{R,x} \quad \vec{F}_{R,y}$$

$$|\vec{F}_R| = \sqrt{F_{R,x}^2 + F_{R,y}^2}$$

Direction

$$\theta = \tan^{-1} \left( \frac{F_{R,y}}{F_{R,x}} \right)$$
Example

The cables attached to the screw eye are subjected to the three forces shown.

Q: Determine the magnitude and angle of the resultant force vector.
(1) Draw FBD

\[ \begin{align*}
\vec{F}_1 &= 300N \\
\vec{F}_2 &= 450N \\
\vec{F}_3 &= 600N
\end{align*} \]

(2) Determine magnitude

How? Resolve each vector into components

\[ \begin{align*}
\vec{F}_1 &= 300 \begin{bmatrix} 0 & 1 \end{bmatrix} N \\
\vec{F}_2 &= 450 \begin{bmatrix} -\cos 45 & \sin 45 \end{bmatrix} N \\
&\quad \text{(or } \cos 135) \\
\vec{F}_3 &= 600 \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \end{bmatrix} N
\end{align*} \]

\[ \vec{F}_R = \begin{bmatrix} 0 \ 0 \end{bmatrix} N \]
Vector notation:

Generally: \( \langle x/\hat{i}, \ y/\hat{j}, \ z/\hat{k} \rangle \)

\( \overrightarrow{F_1} = \langle 0, \ 300 \rangle \) N

\( \overrightarrow{F_2} = \langle -450 \cos 45, \ 450 \sin 45 \rangle \) N

\( \overrightarrow{F_3} = \langle 600(\frac{3}{5}), \ 600(\frac{4}{5}) \rangle \) N

Matlab \(
\Rightarrow \text{replace} \langle \rangle \text{with} \ [ ] \\
\text{Now you get to add it up!}
\)

Matlab \( \text{norm}([\ ] \ []) \) gives magnitude
Cartesian vectors

Rectangular coordinate system: formed by 3 mutually perpendicular axes, the x, y, z axes, with unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ in these directions.

Right-handed coordinate system

Rectangular components of a vector

$$ \mathbf{A} = A_x + A_y + A_z $$

Cartesian vector representation

$$ \mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} $$
Magnitude of Cartesian vectors

\[ A = |A| = \sqrt{A_x^2 + A_y^2 + A_z^2} \]

Direction of Cartesian vectors

Expressing the direction using a unit vector:

\[ \mathbf{u}_A = \frac{A}{A} = 1 \]

Direction cosines are the components of the unit vector:

\[ \cos(\alpha) = \frac{A_x}{A} \]
\[ \cos(\beta) = \frac{A_y}{A} \]
\[ \cos(\gamma) = \frac{A_z}{A} \]
Example

The cables attached to the screw eye are subjected to the three forces shown.

**Q: Determine the magnitude and angle of the resultant force vector**
\[ F_1 = 350 \text{N} \quad F_2 = 100 \text{N} \quad F_3 = 250 \text{N} \]

\[ \vec{F}_1 = \begin{pmatrix} 0 \\ 350 \cos 50 \quad 350 \sin 50 \end{pmatrix} \text{N} \]

\[ \vec{F}_2 = \begin{pmatrix} 100 \cos 45 \\ 100 \cos 60 \\ 100 \cos 120 \end{pmatrix} \text{N} \]

\[ \vec{F}_3 = \begin{pmatrix} 250 \cos 60 \\ -250 \cos 45 \\ 250 \cos 135 \\ 250 \cos 60 \end{pmatrix} \text{N} \]

\[ \vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \]
Example

The cables attached to the screw eye are subjected to the three forces shown.

**Q: Determine the direction cosines of the resultant force vector**
Directions?

Approach? Use direction cosines with \( \vec{F}_R \):

\[
\cos(\alpha_R) = \frac{F_{R,x}}{|\vec{F}_R|}
\]

\[
\cos(\beta_R) = \frac{F_{R,y}}{|\vec{F}_R|}
\]

\[
\cos(\gamma_R) = \frac{F_{R,z}}{|\vec{F}_R|}
\]
Position vectors

A position vector $\mathbf{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example,

$$\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$$

expresses the position of point $P(x,y,z)$ with respect to the origin $O$.

The position vector $\mathbf{r}$ of point $B$ with respect to point $A$ is obtained from

$$\vec{r}_B = \vec{r} + \vec{r}_A$$

$$\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\vec{r} = (x_B \mathbf{i} + y_B \mathbf{j} + z_B \mathbf{k}) - (x_A \mathbf{i} + y_A \mathbf{j} + z_A \mathbf{k})$$

$$\vec{r} = (x_B - x_A) \mathbf{i} + (y_B - y_A) \mathbf{j} + (z_B - z_A) \mathbf{k}$$