To do ...

- HW 25 ME due Thurs

- No class on Friday (Nov 18)!
- No office hours during break

*Different scales but similar shapes!

\[ \text{total length } L > L \]

under weight - catenary

\[ \text{total length } L > L \]

distributed load - parabolic
Cables

Assume:
- Weight is negligible
- Perfectly flexible
- Inextensible

- Loads $> \text{weight}$
- No resistance to bending
- Constant length
Cable subjected to concentrated loads

**Main Ideas:**

1. Horizontal component of tension is constant at every point!

2. $A_x$, $A_y$, $B_x$, $B_y$ are two unknowns because we do not know the sag $y_c$ and $y_d$, therefore we do not know the magnitude or direction at $A$ or $B$.

For any $P_i$, the structure would have the same...
for any combination of $P_1$ and $P_2$, there are infinitely many configurations.

Q: What if $P_2 = 0$?
Determine the tension in each segment of the cable shown.

**Main Idea:**

1. Horizontal component of tension is constant, vertical component depends on slope of segment.

   if $\Theta \uparrow$ then $T \uparrow$
Cable subjected to a distributed load

**KEY Equations:**

\[ \Sigma F_x: \text{ gives } T \cos \theta = \text{constant} = F_{\text{ext}} \]

\[ \Sigma F_y: \text{ gives } T \sin \theta = \int w(x) \, dx \]

\[ \Sigma M: \text{ gives } \frac{dy}{dx} = \tan \theta \]

*Use these to get the cable shape:*
Use these to get the causal surge.

\[ y(x) = \frac{1}{F_0} \int (\int w(x) \, dx) \, dx \]
Determine the maximum force developed in the cable and the cable’s required length. The span length and sag are known.

**DEAN THE FBD**

**START WITH THE CABLE EQUATION**

\[ y(x) = \frac{1}{\pi^2} \int (\sin(x) \cdot dx) \]  
using \( \omega(x) = \omega_0 \)

\[ y(x) = \frac{1}{\pi^2} \int (\omega \cdot dx) \]  
\[ y(x) = \frac{1}{\pi^2} \int (\omega_0 + c_1) \]  
\[ y(x) = \frac{1}{\pi^2} \left[ \frac{\omega_0}{2} x^2 + c_1 x + c_2 \right] \]

Here, \( c_1 \) and \( c_2 \) are integration constants from the indefinite integral.

**USE BOUNDARY CONDITIONS TO SOLVE FOR \( c_1, c_2, \) AND \( F_i \)** (i.e., use points on the cable that you know like \( x=0, y=0 \) )

**USING** \( x=0, \) then \( y(x=0) = 0 \)

\[ 0 = \frac{1}{\pi^2} \left( \frac{\omega_0}{2} (0)^2 + c_1 (0) + c_2 \right) \]

\[ \Rightarrow \quad c_2 = 0 \]

**USING** the slope relation, \( x=0 \) then \( \frac{dy}{dx} \bigg|_{x=0} = 0 \)
\[ 0 = \frac{1}{2} \left( w(x) + c_1 \right) \quad \therefore \quad c_1 = 0 \]

Finally, if \( x = \frac{L}{2} \) then \( y(x = \frac{L}{2}) = h \)

\[ h = \frac{1}{F_h} \left( \frac{w_0}{2} \left( \frac{L}{2} \right)^2 \right) \]

\[ F_h = \frac{w_0 l^2}{8h} \quad \text{horizontal component of tension!} \]

The cable span is now:

\[ y(x) = \frac{w_0 x^2}{2F_h} = \frac{w_0 x^2}{2} \cdot \frac{8h}{w_0 l^2} = \left( \frac{4h}{l^2} \right) x^2 \]

parabolic! and depends only on span and sag!

The tension is given by:

\[ T(x) \cos(\theta(x)) = \frac{w_0 l^2}{8h} \]

Keep in mind that \( T \) and \( \theta \) are functions of \( x \), so actually

\[ T(x) \cos(\theta(x)) = \frac{w_0 l^2}{8h} \]

giving

\[ T(x) = \frac{w_0 l^2}{8h \cos(\theta(x))} \]

Q: What is the range of \( \theta \), or how can we find \( \theta(x) \)?

Using the relation:

\[ \frac{dy}{dx} = \tan \theta \]

\[ \theta(x) = \tan^{-1} \left( \frac{dy}{dx} \right) = \tan^{-1} \left( \frac{dy}{dx} \left( \frac{y(x)}{x^2} \right) \right) \]
\[ \Theta(x) = -\tan^{-1}\left( \frac{8h}{L^2} x \right) \]

This would suggest that \( \Theta_{\text{max}} \) is

\[ \Theta_{\text{max}} (x = \frac{L}{2}) = \tan^{-1}\left( \frac{4h}{L} \right) \]

Finally, the total length is found using the length of a segment:

\[ ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \]

Integrate over the cable:

\[ L = \int ds = 2 \int_0^{\frac{L}{2}} \sqrt{1 + \left( \frac{4h}{L^2} x \right)^2} \, dx \]

\[ L = \frac{L}{2} \left[ \sqrt{1 + \left( \frac{4h}{L} \right)^2} + \frac{L}{4h} \sinh^{-1}\left( \frac{4h}{L} \right) \right] \]

Use this to solve next example!
If the pipe has a mass per unit length of 1500 kg/m, determine the tension developed in the cable.

\[ L = 30 \text{ m} \]
\[ h = 3 \text{ m} \]
\[ w(x) = 1500 \frac{\text{kg}}{\text{m}} \]
\[ g = 9.81 \text{ m/s}^2 \]

*Use MATLAB code to check work!*