To do ...

- Quiz 6 (ends Sat)
- WA12 due Sun
- HW 24 PL due Tues
- HW 25 ME due Thurs
Determine the magnitude and location of the resultant hydrostatic force acting on the surface of a seawall shaped in the form of a parabola. The wall is 5 m. The density of the sea water is 1020 kg/m$^3$.

\[ \omega_B = p_B b = g \cdot g z b \]

\[ F_R = \frac{1}{2} \omega_B z = \frac{1}{2} g b z^2 = 225.1 \text{ kN} \]

\[ \text{Weight} = \text{g} V = \text{g} A \cdot b \]

\[ A = \frac{1}{3} ab \]

\[ W = g g b \left( \frac{1}{2} (1 \times 3) \right) = 99b = 50 \text{ kN} \]

The resultant force is

\[ F_R = \sqrt{F_{h}^2 + W^2} = \sqrt{(225.1)^2 + (50)^2} = 231 \text{ kN} \]
Determine the magnitude of the hydrostatic force acting on gate AB which has a width of 1.5 m.

\[ F_h = \frac{1}{2} \rho gh h = \frac{1}{2} \rho gh^2 \]

\[ F_v = \rho g h l = \rho g lb \frac{1}{2} hw \]

\[ F_v = \frac{1}{2} \rho gbhw \]

\[ \omega_B = \rho b = \rho hb \]

\[ F_r = \sqrt{F_h^2 + f_r^2} = \sqrt{\left(\frac{\rho gbh^2}{2}\right)^2 + \left(\frac{\rho gbwh}{2}\right)^2} \]

\[ F_r = \frac{\rho gbh}{2} \sqrt{h^2 + w^2} \]

\[ a = \sqrt{h^2 + w^2} \]

\[ F_r = \frac{1}{2} \rho gbha \]
The 2-m-wide rectangular gate is pinned at its center A and is prevented from rotating by the block at B. Determine the reactions at these supports due to hydrostatic pressure.

$$f_c = w_c h = pbh = ghbhz_c$$

$$f_B = \frac{1}{2} (w_B - w_c)h = \frac{1}{2} ghbh(z_B - z_c)$$

$$f_F = f_c + f_B = ghbh \left( \frac{z_B + z_c}{2} \right)$$

**Sum the moment about A.**

$$\Sigma M_A: \quad f_B (0.5) - B_x (1.5) = 0$$

$$\frac{3}{2} B_x = \frac{1}{2} f_B \quad \Rightarrow \quad B_x = \frac{1}{3} f_B$$

29.4 kN
Sum the forces in \( x \)

\[ \Sigma F_x: \quad F_c + F_B - A_x - B_x = 0 \]

\[ A_x = F_c + F_B - B_x = 235 \text{ kN} \]
Determine the magnitude of the hydrostatic force acting on gate AB which has a width of 2 m. The specific weight of water is 62.4 lb/ft³.

\[ \text{width } b = 2 \text{ m} \]
\[ \gamma = 62.4 \text{ lb/ft}^3 \]

The horizontal component is:

\[ F_H = F_1 + F_2 = \gamma h (z_2 - z_1) + \frac{1}{2} (z_2 - z_1)(\omega_2 - \omega_1) \]
\[ F_H = p_r b(z_2 - z_1) + \frac{1}{2} (z_2 - z_1) b (p_2 - p_1) \]
\[ \sum \rho \text{ah } z (z_2 - z_1) + \frac{1}{2} \rho a h (z_2 - z_1)(z_2 - z_1) \]
\[ \mathbf{\tau} = \int \mathbf{r} \times \mathbf{F} \, dV \]

\[ \mathbf{F}_h = \mathbf{g}b(x - z_2) \left[ z_1 + \frac{1}{2}(z_2 - z_1) \right] \]

\[ \mathbf{F}_h = \frac{1}{2} \mathbf{g}b (z_2 - z_1)(z_2 + z_1) \]

the vertical component is found using:

**Method I**

\[ \mathbf{F}_v = \mathbf{F}_3 - \mathbf{F}_4 = w_2 x - \mathbf{g}b \mathbf{v} \]

\[ \mathbf{F}_v = p_2 b x - \mathbf{g}b \left( \frac{1}{2} x (z_2 - z_1) \right) \]

\[ \mathbf{F}_v = \mathbf{g}b x (z_2 - \frac{1}{2}(z_2 - z_1)) \]

\[ \mathbf{F}_v = \frac{1}{2} \mathbf{g}b x (z_1 + z_2) \]

the magnitude of the resultant force is:

\[ \| \mathbf{F}_2 \| = \sqrt{\mathbf{F}_a^2 + \mathbf{F}_v^2} \]
\[ \| \vec{F}_r \| = \sqrt{\left( \frac{1}{2} s g b (z_1 + z_2)(z_2 - z_1) \right)^2 + \left( \frac{1}{2} s g b (z_1 + z_2)x \right)^2} \]

\[ \| \vec{F}_r \| = \frac{1}{2} s g b (z_1 + z_2) \sqrt{(z_2 - z_1)^2 + x^2} \]

**Method II**: Use composite areas!

the horizontal component is the same as above.

the vertical component is the weight of "water" above the gate.

\[ F_v = g g V \]

\[ F_r = e a h \left( x_2 + \frac{1}{2} x (z_2 - z_1) \right) \]
\[ \vec{F}_V = ggb \left( xz_1 + \frac{1}{2} \times (z_2 - z_1) \right) \]

\[ \vec{F}_V = \frac{1}{2} ggb x (z_1 + z_2) \quad \text{SAME AS ABOVE!} \]
The factor of safety for tipping of the concrete dam is defined as the ratio of the stabilizing moment due to the dam’s weight divided by the overturning moment about O due to the water pressure. Determine this factor if the concrete has a density of 2500 kg/m$^3$.

- $w_1 = \rho g V = \rho g b \left( \frac{1}{2} (3x_1) \right) = 9 \rho g b$
- $w_2 = \rho g V = \rho g b \left( 1 - 6 \right) = 6 \rho g b$
- $F_R = \frac{1}{2} w_0 \cdot 6 = \frac{1}{2} \rho b \cdot 6$
- $F_R = \frac{1}{2} (6) b \ \rho g (6) = 18 \ \rho g b$

- $x_1 = \frac{2}{3} (3) = 2 \ \text{m}$
- $x_2 = 3 + \frac{1}{2} = 3.5 \ \text{m}$
- $x_3 = \left( \frac{1}{3} \right) 6 = 2 \ \text{m}$

The width $b$ is not given.
The overturning moment about O from the hydrostatic pressure is

\[ N_{ot} = x_3 F R \]

The stabilizing moment from the weight of the concrete dam is:

\[ M_s = x_1 W_1 + x_2 W_2 \]

The factor of safety is:

\[
F.S. = \frac{M_s}{N_{ot}} = \frac{x_1 W_1 + x_2 W_2}{x_3 F} = \frac{x_1 (9.99 b) + x_2 (6.89 b)}{x_3 (18.92 g b)}
\]

\[
F.S. = \frac{(3.9 g b)(6.89 b)}{(3.9 g b)(6.92 x_3)}
\]

\[
F.S. = \frac{5}{3} \left( \frac{13}{12} \right) = \frac{2500}{1050} \cdot \frac{13}{12} = 2.71
\]